HOMEWORK 9

Due: 11/09/2016

1. Textbook Exercises

10.5: 3, 4, 7, 11, 22

10.6: 2, 8, 12, 15

Note: (1) By a "steady state solution", we mean a solution that is independent of t. (2) For question **10.5.7**, work from scratch following *Separation of variables* $\Rightarrow$ *Two-point BVP* $\Rightarrow$ *Formal Fourier series expansion*; and similarly for question **10.6.15**. For the rest of the questions, you may choose to use known formulae, if applicable.

2. Additional Exercises

A1-A4. Problem 74, 75, 76, 78 from Prof. Nolen's *Additional Homework Problems.*

Hints:

A1(No.74): In principle, this is how we solved initial-boundary problem for heat equations.

A2(No.75): Find a steady-state solution to the boundary-value problem. There turns out to be only one other than the constant zero solution.

A3(No.76): Just write down the **initial boundary value problem** satisfied by $v(t,x)$; no need to solve.

A4(No.78): You'll need to figure out what α is in $\alpha^2 u_{xx} = u_t$ satisfied by the given $w(t,x)$. Moreover, if you are interested, you may find out that $E(t) := \int_{-\infty}^{\infty} w(t,x)dx$ equals to a fixed constant for all t. This is not surprising since, in this case, one expects the total amount of heat along the infinite rod to be a conserved quantity over time. For those who are curious, a little trick for computing $I = \int_{-\infty}^{\infty} e^{-x^2}dx$ is taking the product

$$I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2}dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2}dy\right) = \int_{\mathbb{R}^2} e^{-(x^2+y^2)}dxdy,$$

then turn to polar coordinates:

$$I^2 = \int_0^{2\pi}\int_0^{\infty} e^{-r^2} r\, drd\theta.$$