HOMEWORK 6

Due: 10/19/2016

1. Textbook Exercises

6.1: 3, 6, 9, 26, 27

6.2: 3, 8, 9, 14, 16

6.3: 6, 14, 16, 17, 21, 33, 37

Comment: Of course, a Laplace transform can be computed from the scratch using its very definition, but using known facts such as the *first and second shifting formulae* and the *derivative formulae* can quite often simplify calculation. There's not much theory in here, just practice, and pick up some sensitivity.

2. Additional Questions

A1. Suppose that

$$\mathcal{L}\{f(t)\} = F(s).$$

Use the definition of the Laplace transform to derive a formula for $\mathcal{L}\{f(at)\}$, where $a > 0$ is a constant. Why do we require that the constant a is positive? How would you test the correctness of your formula (Optional)?

A2. Use the derivative formula

$$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$$

to show that

$$\mathcal{L}\left\{\int_0^t f(x)dx\right\} = \frac{\mathcal{L}\{f(t)\}}{s}.$$

Use this latter formula to compute $\mathcal{L}^{-1}\{s^{-2}\}$, then compare with what you learned before about $\mathcal{L}\{t\}$.

A3. Use the Laplace and inverse Laplace transforms to solve the second order differential equation

$$y'' - (a+b)y' + (ab)y = 0$$

with initial values set to be $y(0), y'(0)$, where a, b are arbitrary real constants.