HOMEWORK 1

Due: 09/05/2016

1. Textbook Exercises

Note: Though not every problem listed on the syllabus is required for turning-in, you are expected to be able to solve each of them. My advice is that you start early, say, after each class, working on relevant problems in the book. When the homework file is released, set aside a reasonable amount of time, say, 10 minutes for each question, and neatly write down your answers.

2.1: 20, 28, 33
2.2: 3, 7, 10, 21, 31
2.3: 9

2. Additional Questions

A1. For each of the following equations, name the method you plan to use to find solutions. Then choose two (or more) and actually solve them.

$$(a) \quad y((y')^2 + 1) = 1$$
$$(b) \quad x^2y' = 3xy - 2y^2$$
$$(c) \quad (\sin x)y' = -(\cos x)y + 2x$$
$$(d) \quad xy' = y + 2xe^{-y/x}$$

A2. Pick a differentiable function $y = f(x)$, then give two distinct ODEs that are both solved by $y = f(x)$.

A3. Here is another example in which substitution helps finding solutions. Consider the *Bernoulli's equation* (the same term appears in fluid dynamics, but with different content):

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

(a) Verify that the population model equation with reproduction rate r and environmental capacity C,

$$\frac{dp}{dt} = rp\left(1 - \frac{p}{C}\right),$$

is a special case of the Bernoulli's equations.

(b) For $n \neq 0, 1$, the Bernoulli equation is nonlinear. Show that the the substitution $z = y^{1-n}$ transforms the equation into a linear equation in z.

(c) Use the idea in part (b) to solve the equation

$$xy^2\frac{dy}{dx} + y^3 = x.$$