Math 3430-03 Spring 2020
Practice Exam I

I have neither given nor received any unauthorized help on this exam and I have conducted myself within the guidelines of the CU Community Standard.

Name: ________________________ Signature: ________________________

Instructions:

- You may use a formula sheet, A4 size, front and back, prepared on your own. Otherwise, notes, books, calculators or computers are not allowed in this exam.
- Make sure you show the work that leads to your answer to receive full credit. If you are using a theorem or a fact to draw some conclusions, quote the result.
- This test contains **5 questions**. You have **50 minutes** to answer all the questions.

Good Luck !

Question	**Score**
1	/ 20
2	/ 20
3	/ 20
4	/ 20
5	/ 20
Total	**/100**

1. First-order ODEs

1. Solve the following 1st-order initial value problem:

$$(y\cos x + e^{2y}) + (\sin x + 2xe^{2y} + 2y)\frac{\mathrm{d}y}{\mathrm{d}x} = 0, \qquad y(0) = 1.$$

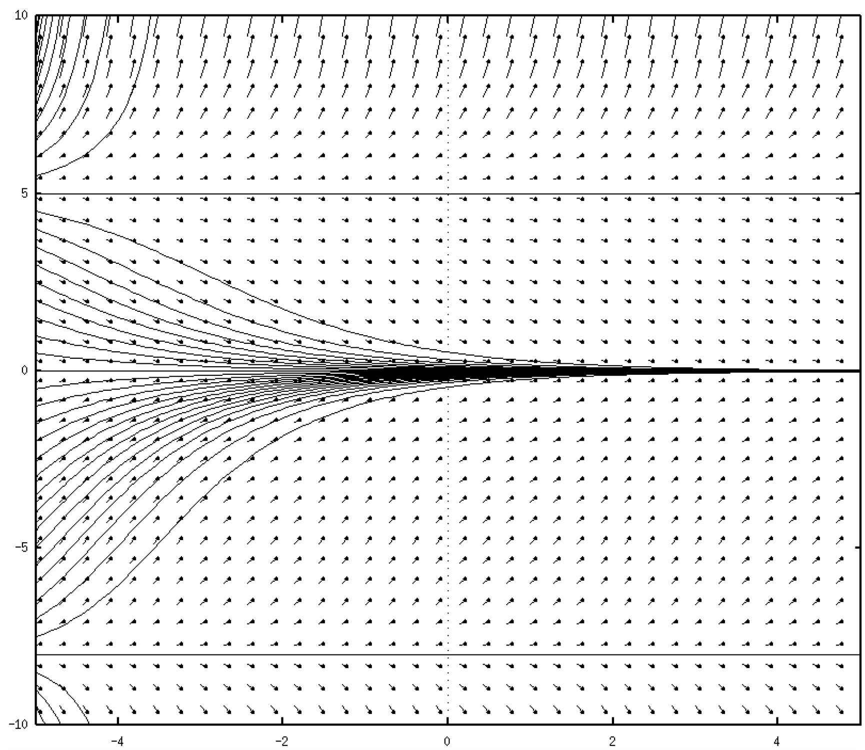
2. Find the general solutions of the following 1st-order ODE:

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = x^4.$$

3. This question is concerned with "direction fields".

(1) What's special about the direction field of an **autonomous** first-order ODE?

(2) Write down a 1st-order ODE $y' = f(y)$ that has the following direction field (with various integral curves). (Note: The three horizontal integral curves are at $y = -8, 0, 5$, respectively.)



2. Higher-order Constant Coefficient ODEs

4. Find a **basis** for the solution space of the following 6th-order ODE:

$$y^{(6)} + 7y^{(5)} - 6y^{(4)} - 70y''' - 235y'' - 309y' - 252y = 0$$

You may find this formula useful:

$$\lambda^6 + 7\lambda^5 - 6\lambda^4 - 70\lambda^3 - 235\lambda^2 - 309\lambda - 252$$
$$= (\lambda^2 + 2\lambda + 3)^2(\lambda + 7)(\lambda - 4).$$

3. Applications

5. Let C_1 be a 1-parameter family of curves that satisfy the equation

$$y^3 = cx^5,$$

where c is the parameter. Now establish a differential equation for its orthogonal trajectories C_2 and then solve the equation.