Math 3430-03 Spring 2020
Exam I

I have neither given nor received any unauthorized help on this exam and I have conducted myself within the guidelines of the CU Community Standard.

Name: ________________________ Signature: ________________________

Instructions:

- You may use a formula sheet, A4 size, front and back, prepared on your own. Otherwise, notes, books, calculators or computers are not allowed in this exam.
- Make sure you show the work that leads to your answer to receive full credit. If you are using a theorem or a fact to draw some conclusions, quote the result.
- This test contains **5 questions**. You have **50 minutes** to answer all the questions.

Good Luck !

Question	**Score**
1	/ 20
2	/ 20
3	/ 20
4	/ 20
5	/ 20
Total	**/100**

1. First-order ODEs

1. Solve the following 1st-order initial value problem:

$$(y^2 e^{xy^2} + 3x^2) + (2xye^{xy^2} + \cos y)\frac{\mathrm{d}y}{\mathrm{d}x} = 0, \qquad y(0) = \frac{\pi}{2}.$$

2. Find the general solutions of the following ODE:

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \frac{\sin x}{x}, \quad x \neq 0.$$

3. This question is concerned with 1st order homogeneous equations.

(1) A 1st-order ODE of the form

$$y' = f(x, y),$$

is said to be **homogeneous** if $f(x, y)$ is *homogeneous of order* 0. What does "homogeneous of order 0" mean?

(2) What's special about the direction field of a **homogeneous** first-order ODE?

(3) Solve the following 1st-order initial value problem and specify the open interval (a, b) on which your solution is defined.

$$(xy + y^2) - x^2 \frac{\mathrm{d}y}{\mathrm{d}x} = 0, \qquad y(1) = 1.$$

2. Higher-order Constant Coefficient ODEs

4. Find a **basis** for the solution space of the following 6th-order ODE:

$$y^{(6)} - 12y^{(5)} + 35y^{(4)} + 31y''' - 90y'' - 175y' - 750y = 0$$

You may find this formula useful:

$$\begin{aligned}&\lambda^6 - 12\lambda^5 + 35\lambda^4 + 31\lambda^3 - 90\lambda^2 - 175\lambda - 750\\ &= (\lambda^2 + \lambda + 3)(\lambda - 5)^3(\lambda + 2).\end{aligned}$$

3. Applications

5. Let C_1 be a 1-parameter family of curves that satisfy the equation

$$\sin y = ce^{x^2},$$

where c is the parameter. Now establish a differential equation for its orthogonal trajectories C_2 and then solve the equation.