Math 3001-01 Spring 2020
Practice Exam I

I have neither given nor received any unauthorized help on this exam and I have conducted myself within the guidelines of the CU Community Standard.

Name: ________________________ Signature: ________________________

Instructions:

- Notes, books, calculators or computers are not allowed in this exam.
- Make sure you show the work that leads to your answer to receive full credit. If you are using a theorem or a fact to draw some conclusions, quote the result.
- This test contains **4 questions**. You have **50 minutes** to answer all the questions.

Good Luck !

Question	**Score**
1	/ 25
2	/ 25
3	/ 25
4	/ 25
Total	**/100**

1. This question is concerned with the "Archimedean Property".

(1) Write down the content of the **Archimedean Property** of real numbers.

(2) Use the Archimedean Property to prove that

$$\bigcap_{n=1}^{\infty} \left(0, \frac{1}{n}\right) = \emptyset.$$

(3) Another consequence of the Archimedean Property is that both $\mathbb{Q}$ and $\mathbb{I}$ are (fill in the box below with a suitable word)

$$\boxed{}$$

in $\mathbb{R}$.

2. This question is concerned with "cardinality".

(1) Let A, B be two sets. What does it mean by saying that A and B have the **same cardinality**? (Write down the definition.)

(2) Use the definition to prove that the closed interval $(0, 1)$ and $\mathbb{R}$ have the same cardinality. (You need to provide a single function that meets the requirements in the definition.)

3. This question is concerned with the "Monotone Convergence Theorem".

(1) State the **Monotone Convergence Theorem** of sequences.

(2) Define a sequence (a_n) by

$$a_1 = \sqrt{2}, \quad a_{k+1} = \sqrt{2 + a_k} \quad (k = 1, 2, \ldots).$$

Use mathematical induction to show that (a_n) is increasing.

(3) Is the sequence in part (2) convergent? If so, what is the limit? Justify your answer.

4. For each of the following, either give an example that satisfies the condition(s), or explain why an example does not exist.

(1) A sequence $(a_n)_{n=1}^{\infty}$ with $\lim_{n\to\infty} a_n = 0$ and $\sup\{a_n\}_{n=1}^{\infty} = 1$. (Note: $\{a_n\}_{n=1}^{\infty}$ is the **set** formed by all distinct numbers that occur in (a_n).)

(2) A convergent sequence (a_n) in which both 0 and 1 occur infinitely many times.

(3) A sequence of **closed intervals** I_n satisfying $I_{n+1} \subseteq I_n$ for all $n \in \mathbb{N}$ and

$$\bigcap_{n=1}^{\infty} I_n = \{2020, 2019\}.$$