Math 3001-01 Spring 2020
Exam I

I have neither given nor received any unauthorized help on this exam and I have conducted myself within the guidelines of the CU Community Standard.

Name: ________________________ Signature: ________________________

Instructions:

- Notes, books, calculators or computers are not allowed in this exam.
- Make sure you show the work that leads to your answer to receive full credit. If you are using a theorem or a fact to draw some conclusions, quote the result.
- This test contains **4 questions**. You have **50 minutes** to answer all the questions.

Good Luck !

Question	**Score**
1	/ 25
2	/ 25
3	/ 25
4	/ 25
Total	**/100**

1. This question is about the **supremum** of subsets of $\mathbb{R}$.

(1) Let $A \subset \mathbb{R}$ be a nonempty subset. As long as A is (write down a suitable word in each of the two boxes below)

$$\boxed{}\ \boxed{},$$

there exists a number $s \in \mathbb{R}$ that satisfies the following two conditions:

i.

ii.

Such an s is called the **supremum** of A; and the fact described above is called the

$$\text{Axiom of } \boxed{}.$$

(2) Determine the **suprema** of the following subsets of $\mathbb{R}$, if they exist.
(You do *not* need to prove your conclusions.)

i. $\left(0, \sqrt{3}\right) \cap \mathbb{Q}$

ii. $\displaystyle\bigcap_{n=1}^{\infty} \left(-\infty, \frac{2}{n}\right)$

iii. $\displaystyle\bigcup_{n=1}^{\infty} (n, n+1)$

2. This question is concerned with "cardinality".

(1) Let A be a set. What does it mean by saying that A is **countable**? (Write down the definition.)

(2) Cite a known property of countable sets to prove that the Cartesian product $\mathbb{N} \times \mathbb{N}$ is countable.

3. This question is concerned with the "Monotone Convergence Theorem".

(1) State the **Monotone Convergence Theorem** of sequences.

(2) Define a sequence (a_n) by

$$a_1 = 1, \quad a_{k+1} = \frac{1}{5 - a_k} \quad (k = 1, 2, \ldots).$$

Use mathematical induction to show that (a_n) is decreasing.

(3) Does the sequence in part (2) converge? If so, what is the limit? Justify your answer.

4. For each of the following, either give an example that satisfies the condition(s), or explain why an example does not exist.

(1) A divergent sequence (a_n) all whose convergent subsequences share the same limit.

(2) A convergent sequence (a_n) that is **not** monotone.

(3) A sequence (a_n) that satisfies: $0 < a_n < 2020$ for all $n \in \mathbb{N}$, and all subsequences of (a_n) diverge.