HOMEWORK QUIZ 1

Due: Friday, Feb.3

1. Supposing that A and B are two events in a random experiment. For each of the following two settings, determine whether A and B are independent, depend, or there is insufficient given information.

(1) $\mathbb{P}(A) = 0.6$, $\mathbb{P}(B) = 0.8$;
(2) $\mathbb{P}(A) = 0.8$, $\mathbb{P}(B) = 0.2$, $\mathbb{P}(\text{not } A \text{ nor } B) = 0.16$.

2. Assume that you are a game provider, the game being simply rolling a die once, then flipping a coin once. Both the die and the coin are fair. If the coin turns out "head", the gamer is rewarded x dollars, where x is the number showing on the die; otherwise, the gamer receives nothing. Write down the sample space. Let X be the random variable standing for the reward to the gamer after one game. What is the value of $\mathbb{E}[X]$? How much would you charge for each play in order to make an expected profit of \$1 per game?

3. For which values of x does the series $\sum_{n=1}^{\infty} e^{nx} = e^x + e^{2x} + e^{3x} + ...$ converge?

4. Given that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... = \frac{\pi^2}{6}$, what is the limit of the series $\sum_{n=1}^{\infty} \frac{1}{(2n)^2} = \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + ...$? Then, what is the limit of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$?

5. Let $\{a_n\}_{n=1}^{\infty}$ be a convergent *sequence* of real numbers, that is, there exists a (finite) real number A, such that $\lim_{n\to\infty} a_n = A$. Show that the series $\sum_{n=1}^{\infty}(a_{n+1} - a_n)$ converges by analyzing the limiting behavior its N-th partial sum. What is the limit of this series? (Note: See, for example, what the series looks like when $a_n = \frac{1}{n}$, or when $a_n = 7^{\frac{1}{n}}$, etc.)

6. Use the comparison test to show that the series $\sum_{n=2}^{\infty} \frac{5+\sqrt{n}}{n^2+1}$ converges.

7. Use the integral test to show that $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ diverges.

8. What is the convergence of the series $\sum_{n=1}^{\infty} \frac{n^3}{e^n}$?

9. What are the lower and upper bounds incurred by taking the first 10 terms of the convergent series $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$?

Bonus. Does the series $\sum_{n=1}^{\infty} \sin n$ converge? Give your reasoning.

Relevant text pages or hints:

1. CoursePack pp.140-141: independence of random events.
2. CoursePack p.145: random variables; coursePack p.153: expectation.
3. CoursePack p.163: geometric series.
4. CoursePack p.167: theorem 1.
5. CoursePack p 162: N-th partial sum; pp.163-164: "telescoping" series.
6. Note that for $n > 25$, one has $5 < \sqrt{n}$; also note that $1 + n^2 > n^2$.
7. Note that $\frac{d}{dx} \ln(\ln x) = \frac{1}{x \ln x}$ $(x > 0)$.
8. Ratio test.
9. CoursePack pp. 178-179. Error bound with integral test.
Bonus. n-th term test, but you'll need to be able to convince yourself that, for example, $\sin n$ attains values $> \frac{1}{2}$ infinitely many times; this latter argument being a little outside the scope of the current course, but not impossible if you realize that $7 - 2\pi \approx 0.72 < 2\pi/3$.