To start from an fc word w, right justify it, and produce the factors of the canonical basis elt, the two key technicalities are:

(1) Make sure every internal letter keeps both its lateral witnesses immediately to its left and right, except for those internal b_2's witnessed in R by a bilateral b_1 on the right. necessary to ensure this.

(on the other hand, if we get a $|12$-string $|2$ where the 2 is not internal, we should release/yield the 1 to the left to accompany any internal 2 it may witness. e.g. $|\bar{2}3|24$.)

(2) Telling $s_2\overline{s_1 s_2}$ apart from $s_2\bar{s}_1 s_2$ to make the correct f-factor assignment.

difference: whether there's a 1 on the right

Thus, we'll use two trackers along w/ 12-coset decompositions:

(1) yield, either (1,) or (,) ;

to do — not to do

(2) one_on_right, either True or False

Note: ⓐ We can yield the 1, i.e., we have yield = (1,), only if one_on_right = True.

ⓑ We'll only yield a 1 on the very left of a coset parabola part if the parabola is 1 or 12 where 2 is not internal,

ⓒ By ⓑ, a 1 yielded to the left can be kept on the left.

The algorithm:

$w := x_2 \mid y$ x ends w/ rightmost 2 in w

$1 \in y$

yield = (1)

one-on-right = True

e.g. $w = 231$ can think of 1 as a fine coset dec. parabolic (so yield it)

$x' = x1$, $y = 1y'$,

record the letters in y' to factors, x_I

12-decompose x'

x'_I: not internal

21 $\bar{2}1\bar{2}1$ $1\bar{2}1$

$x'_I \rightsquigarrow$: $b_2 b_1$ $b_2 b_1 b_2 b_1 - 2 b_2 b_1$ $b_1 b_2 b_1 - b_1$

(b)

don't yield 1

x'^I : no 1 or 2. easy

$1 \notin y$

yield = (,)

one-on-right = False

12-decompose x

2 $2\bar{1}2$ not int 12 $\bar{1}\bar{2}12$

$x_I \to$ b_2 $b_2 b_1 b_2 - b_2$ yield 1 (by (b)) $b_1 b_2 b_1 b_2 - 2 b_1 b_2$

no 1 or 2, easy

x^I

record only b_2, (record the yield b_1 later.)

(b)

don't yield

set one-on-right to True.

run recursion w/ $w = x'^I / x^I$.