How to right justify an fc word (in type H)

Def. from Green's paper.

$b = b_{i_1} b_{i_2} \cdots b_{i_k}$ where $w = i_1 i_2 \cdots i_k$ is an fc word w.

Take b_p. Then b_p is

— *internal* if w has a reduced word $\cdots b_q b_p b_q$ w/ $m(p,q) \geq 3$. (type H: $p \in \{1,2\}$; type B: $p \in \{1,2\}$);

otherwise, b_p is *external*.

w = 1212.

— *lateral* if b_p is external and w has a red word $b_p b_q$ or $b_q b_p$ w/ b_q internal and $m(p,q) \geq 3$; if in addition b_p can be 'lateral in two ways' / two $b_p b_q b_p$

convex chains in $H(w)$ witness b_p as an ~~extremal~~ side vertex

$b_p \cdots \underline{\underline{b_q}} \cdots (b_p) \cdots \underline{\underline{b_q}} b_p \cdots$, then (this) b_p is bilateral.

different internal b_q's.

(Note: B & H : lateral $\Rightarrow$ is $\{b_1, b_2\}$)

Lemma 3.2.1. In type H, a bilateral letter must be b_1.

(In type B. ?)

Ex. (H) $b_1 \underline{\underline{b_2}}\, b_3 (b_1) \underline{\underline{b_2}}\, \underline{\underline{b_1}}\, b_2$.

lateral, not bil.

bil

lateral, not bil.

internal

Def. The set R for $\underline{w} = b_{i_1} \cdots b_{i_q}$ is the set of letters s s.t.

① s is internal. → have code for checking this.

② after commutations, we can move it to a subsequence

$t\,\underline{s}\,\underline{\underline{t}}$ where $\underline{\underline{t}}$ is bilateral.

($\underline{s}$ ↓ our s)

$\underline{w} = b_1\, b_6\, \underline{b_2}\, b_5\, \underline{\underline{b_1}}\, b_3\, b_2\, b_1$

Def. $\underline{w}$ is <u>right justified</u> iff

✓ (1) $\forall s \in R$, the letter imm. to the left of s is <u>either lateral or internal</u> (either 1 or 2). → <u>internal or lateral.</u>

✓ (2) for all internal letters s' not in R, both neighbors of s' are

~~Conjecture~~.

Proved false later. see rj.pdf.

(We'll collect the final result by repeatedly prepending things to an initial empty word.)

To right justify an FC word $\underline{w}$, it suffices to

(1) (a) Separate $\underline{w}$ into two parts

$$\underline{w} = x \cdot y$$

so that x ends in 1 or 2 and y contains no 1 or 2.

Then (b) do a $\{1,2\}$ coset decomp. on x to get

$$\underline{w} = x \cdot y = x^{I} \cdot x_{I} \cdot y$$

(2) Prepend $x_I \cdot y$ to the collected word (empty at this point)

(3). Recurse w/ x^I in place of $\underline{w}$, i.e., $RJ(\underline{w}) = RJ(x^I)\cdot[x_I y]$, until x^I is empty.

e.g. $\underline{w} = 16251321$. $\xrightarrow[\text{(1) (a) does nothing}]{(1)}$ $16253 \cdot [121 \cdot (\ )] \longmapsto 6[12 \cdot 53[121]] \longmapsto 61253121.$

$x^I \cdot x_I \quad y$

$6 \to (\ )\cdot(\ )\cdot 6$

Problem: Recall that if (W, S) is a finite Coxeter system w/ longest elt w_0. Then

$$T_{w_0} \cdot C_w = \eta_w C_{\lambda(w)} + \sum_{y \in Y} \alpha_y C_y$$

where $\eta_w \in \{-1, +1\}$, $\lambda(w) \sim_L w$ and all elts in Y are in strictly lower two-sided cells than w, i.e. $y \leq_{LR} w$ but $w \not\leq_{LR} y$. Moreover, $\lambda(\lambda(w)) = w$.

What's η_w? $\lambda(w)$?

Todo (immediate) Compute $T_{w_0} \cdot C_w$ in the KL basis

$\downarrow$ W.long_element()

- $T_{w_0} = T_{s_1} \cdots T_{s_k}$ if $w_0 = s_1 \cdots s_k$ reduced.

$= (C_{s_1} - v^{-1}) \cdots (C_{s_k} - v^{-1})$

- $C_s \cdot C_w = \begin{cases} (v+v^{-1}) C_w & \text{if } sw < w \\ C_{sw} + \sum_{\substack{sy<y<w}} \mu(y,w) C_y & \text{if } sw > w \end{cases}$

s_times_w

( ) Bruhat order, sage has bruhat intervals.

$\mu(y,w)$: use Coxeter3, see hecke.py

Assignments:

Gp 1. Chase, Natalie. code $T_{w_0} \cdot C_w$, maybe also η_w, $\lambda(w)$. assume $LC(w)$ = left star closure of w.

Gp 2. Joel, Thomas. write a proof for the conjecture. finish the code for types B and H.