Last time:

$$TLQ(A_n) \cong TLP_\delta(A_n) \cong TLD_\delta(A_n)$$

$$TLQ(A_n) = H(A_n) / \langle C_{i(i+1)i} \rangle$$

$$TLP_\delta(A_n) = \left\langle E_i \;\middle|\; \begin{array}{l} E_i^2 = \delta E_i \\ E_iE_j = E_jE_i \text{ if } |i-j|>1 \\ E_iE_jE_i = E_i \text{ if } |i-j|=1 \end{array} \right\rangle$$

$$TLD_\delta(A_n) = \langle U_i \mid \dots \rangle$$

Note: The relations in $TLP_\delta(A_n)$ 'come from' relations in Hecke algebras for dihedral systems.

- $E_i^2 = \delta E_i \leftarrow C_i C_i = (v+v^{-1}) C_i$ in H and hence in TLQ

  i.e. $[C_i]\cdot[C_i] = (v+v^{-1})[C_i]$

- $E_iE_j = E_jE_i$ if $m(i,j)=2 \leftarrow C_iC_j = C_jC_i$ in H, hence

  $[C_i][C_j] = [C_j][C_i]$ in TLQ if $m(i,j)=2$.

- (Most interesting)

$$E_iE_jE_i = E_i \text{ if } m(i,j)=3 \longleftarrow C_{iji} = C_iC_jC_i - C_i \text{ in } H,$$

so $[C_i][C_j][C_i] = [C_i]$ in TLA if $m(i,j)=3$.

Rmks:

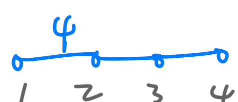
$$C_{1212} = C_1 \cdot C_2 \cdot C_1 \cdot C_2 - 2C_1 \cdot C_2$$

$C_{12} = C_1 \cdot C_2$

$C_2 \cdot C_{12} = C_{212} + C_2$

$C_1 \cdot C_{212} = C_{1212} + C_{12}$

- In general, even outside type A, we can use the same principle to create $TLP(W,S)$: set up generators E_s corr. to KL gen C_s $\forall s \in S$. find relations for $\{E_s\}$ by considering dihedral subsystems of $H(W,S)$.

e.g. B_4 

$\rightsquigarrow$ $TLP(B_4) = \langle E_1, E_2, E_3, E_4 \mid$ same quad. rel. same comm. rel, + new rels for edges $\rangle$

edge: $\{i,j\} = \{1,2\}$ $\longrightarrow$ $E_i E_j E_i E_j = 2 E_i E_j$

$i - j$

because $C_{ijij} = C_i C_j C_i C_j - 2 C_i C_j$

other edges $\longrightarrow$ $E_i E_j E_i = E_i$ as before.

Ex. Find the presentation for $TLQ(H_n)$ H_n: 

- In principle, we know the TLP presentation for every Coxeter system we'll be concerned with.

- On the other hand, it's not trivial to have a diagram realization of $TLQ(W,S) \cong TLP(W,S)$ for general Coxeter systems.

In fact, the realization is only known to exist in types

A, B, H, D, E, $\tilde{C}_n$ ().

(B, H, D, E, $\tilde{C}_n$: hard)

↙

In each case, the diagram realization can be described as a diagram algebra w/ a basis given by noncrossing pairings satisfying certain conditions and w/ mult given by 'stacking + local reduction'.

e.g. B_n : 'some arcs in the diagrams may now carry a certain decoration, but only arcs exposed to the west can + more technical conditions'.

It's in general nontrivial to show that diagrams fitting the description are gen by a obvious generating set coming from KL theory.

It's also in general difficult to find the local reduction rules to reflect the rels in $TLP(W, S)$.

- It's true that for any (W, S), $TLQ(W,S) \cong TLP(W,S)$ have bases indexed by the fully commutative elts in W. When $TLD(W,S)$ exists, there's a bijection between $FC(W)$ and the diagram basis elts.

<u>Q:</u> What 'should be' the diagram for each $w \in FC(W)$?