

MATH 4140/5140. HOMEWORK 8  
Due Wednesday, March 30

**Note:** All numbered exercises are from Erdmann–Holm ([EH]).

- (1) Read Sections 4.1 and 4.2 of [EH].
- (2) Let  $A$  be a  $k$ -algebra and let  $V$  be an  $A$ -module. Prove the following statements, which are often interpreted as “quotients and homomorphic images of modules are the same things”:
  - (a) Let  $\phi : V \rightarrow W$  be an  $A$ -module homomorphism, then  $\text{im}(\phi)$  is isomorphic, as an  $A$ -module, to a quotient module of  $V$ .
  - (b) Let  $U$  be a submodule of  $V$ . Then there is an  $A$ -module  $W$  and a module homomorphism  $\varphi : V \rightarrow W$  such that  $V/U$  is isomorphic, as an  $A$ -module, to  $\text{im}(\varphi)$ .
- (3) Exercise 4.4.
- (4) Exercise 4.5.