MATH 4140/5140. HOMEWORK 7

Due Wednesday, March 9

Note: All numbered exercises are from Erdmann–Holm ([EH]).

(1) Review Chapters 1–3 of [EH] up to (and including) Section 3.2.

(2) Exercise 3.11.

(3) Exercise 3.12.

(4) Let A be a k-algebra and let V be A-module. We say V is *indecomposable* if $V \neq 0$ and V cannot be written as a direct sum $V = M \oplus N$ of two nonzero submodules (in other words, if $V = M \oplus N$ then $M = 0$ or $N = 0$).

   (a) Show that every simple A-module is indecomposable.

   (b) Let $A = k[x]$, let $V = k^2$, and let V_α be the A-module whose underlying vector space is V and where x acts as the map $\alpha \in \text{End}(V) = M_2(k)$ given by the matrix

   $$\alpha = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}.$$

   Show that V is indecomposable but not simple. Note that this example shows that the converse of the statement in (a) is false.