MATH 4140/5140. HOMEWORK 3
Due Wednesday, February 2

Note: All numbered exercises are from Erdmann–Holm ([EH]).

Let k be a field below.

(1) Exercise 1.6.

(2) Exercise 1.18.

(3) Exercise 1.22. (A *division algebra* is an algebra where every nonzero element is invertible multiplicatively.)

(4) Let A be a k-algebra and let I be a two-sided ideal of A. Prove that the natural projection map $\pi : A \to A/I$ given by $\pi(a) = a+I$ for all $a \in A$ is an algebra homomorphism.

(5) Prove the second isomorphism theorem for algebras (as stated in the notes of Lecture 7). You don't need to restate the theorem, and you may use the first isomorphism theorem without proof, but make sure you prove both parts of the second isomorphism theorem.

(6) Recall the notion of *principal ideals* in an algebra from Example 1.19 in [EH]. Prove that in the polynomial algebra $k[x, y]$ the ideal $I := \langle x, y \rangle$ generated by the elements x, y is not a principal ideal.