AA2. Lecture 40.

04. 27. 2022.

Last time:

- finished the proof of Maschke's Theorem
- numerical deductions for the A.W. decomp. of s.s. gp algebras.

Today: Review

Final Exam: take-home, cumulative available 5pm Apr 30 → 5pm May 1 on Canvas
closed-book.

Review / Key topics by Chapter :

Ch1 : · algebras , group algebras , path algebras , polynomial algebras .
(algebras) (and their quotients)

· subalgebras and ideals, quotient algebras

· algebra homs , iso theorems for algebras , the correspondence thm.

Ch2 : · modules and representations (and their equivalence , Thm 2.33) ,
(modules) examples of modules , e.g $M_n(k) \cong k^n$.

· sub/quotient modules

· iso / correspondence thm (s) for modules.

Ch3 : · simple modules , composition series , length of modules
(simplicity)

Ch3, continued: · (results concerning simples / comp series)

~~that lemma~~ Lemma 3.3 — strong cyclicity test for simple modules

the Jordan–Hölder theorem

Schur's Lemma

· examples of simple modules: simples of $M_n(k)$, "$k[x]/\langle f\rangle$", path algebras. (acyclic quivers)

Ch4: · three equivalent def. of s.s. modules, s.s. algebras.

(semisimplicity) · closure properties of s-s modules and s.s. algebras.

· Jacobson radicals, nilpotent ideals, annihilators of modules.

→ that long theorem about Jacobson radicals (of fin-length algebras)

· semisimplicity / non-semisimple of $M_n(k)$, "$k[x]/\langle f\rangle$", path algebras

Ch 5 :

(A.W. Theorem)

- "routine" constructions and proofs
  - opposite algebras, the algebra Λ, etc.

    Given a map $f: A \to B$ with A, B, f specified,

    can you check things like "f is an alg/module iso"?

- The statement of the A.W. Thm in full detail, including Cor. 5.11, significance of "$r, n_1, \dots, n_r$", etc.

- A.W. decomps of "$k[x]/\langle f \rangle$" and path algebras when they are s.s.

Ch 6 (Sections 6.1 – 6.3) :

(Maschke's Thm)

- statement of Maschke's Thm

- numerical deductions for A.W. decomp of s.s. gp algebras. (Corollary 6.8)

Thank you!