

MATH 8174. HOMEWORK 1
Due Friday, September 4

Let k be an arbitrary field unless otherwise specified. Read the first two chapters of Humphreys and complete the following problems.

- (1) Humphreys 1.3.
- (2) Humphreys 1.4.
- (3) Find a basis for the orthogonal algebra D_l and deduce $\dim D_l$.
- (4) Show that, up to isomorphism, there are exactly two Lie algebras of dimension 2 over k .
- (5) Let \mathfrak{g} be the Lie algebra over k with basis $\{a, b, c\}$ and with the Lie bracket determined by $[a, b] = c$, $[b, c] = a$ and $[c, a] = b$.
 - (a) Show that \mathfrak{g} is isomorphic to $\mathfrak{sl}_2(k)$ if $k = \mathbb{C}$.
 - (b) Show that \mathfrak{g} is not isomorphic to $\mathfrak{sl}_2(k)$ if $k = \mathbb{R}$.
- (6) Show that the normalizer of a subalgebra of a Lie algebra \mathfrak{g} is a subalgebra of \mathfrak{g} .
- (7) Humphreys 2.2.
- (8) Erdmann–Wildon 1.19 (Leibniz rule).