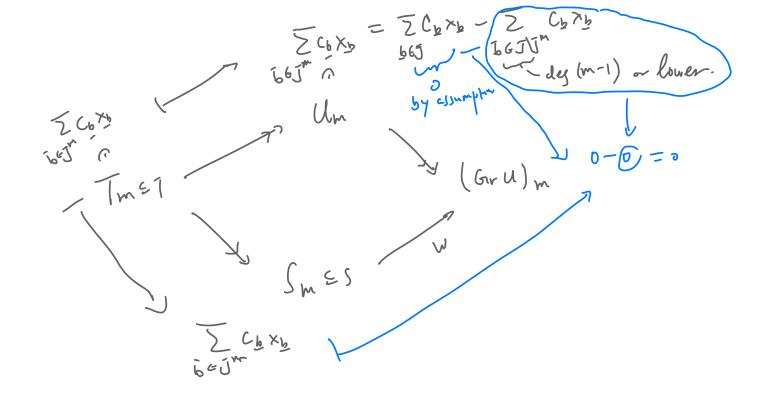
Left time: Two versions of the PBW chemican.
That. The map to from
$$\begin{aligned} &\sum_{m} \left(T_{m} - \frac{1}{p_{m}} - \frac{1}{p_{m}} \right) & (Gr W)_{m} = \frac{1}{q_{m}} \left(\frac{1}{p_{m}} - \frac{1}{p_{m}} \right) & (Gr W)_{m} = \frac{1}{q_{m}} \left(\frac{1}{p_{m}} - \frac{1}{p_{m}} \right) \\ &= S(L) & II \\ &= S(L) \\ &= S(L$$

Consequences of the theorem.

(b) let H be a Lie subalgebra of L. Then the inclusion H c> L induces an inclusion U(H) c> U(L), and U(L) is a free U(H)-produle. free-nodule: take a basis Pf Sketch: $H \longrightarrow U(H)$ $\{X_{ij}\}_{i \in I}$ of H, order L. L'L KIL) then extend the bass to a barrs $\{\chi_i \mid i \in I'\}$ I $\subseteq I' \neq L$. and extend the order to an order on I' with Xi < Xj & i < I - j < I' \ I. ... D

Lass time we observed:

$$P = \frac{1}{2} \operatorname{Thr} 1 = \operatorname{Thr} 2 : Assume Thm 1. i.e., that will on is.
By @, to check β is a basin it remains to check that β is β for β.
ind: Take a lim canb β Cb β = 0$, J a finite set of nonine. I-seq.
We need to show β = 0 β $\$$



Pf that Then 2 => Then 2: Hw. may need something similar to 3 for maps.

Bergman's Dicmond Lemma. general subup - start with a ret X of lettery -ward a basis for a quotient of R == k<X7. the free algebra on (X7 by a contain rdeal I - need a "semigroup partial ordering", a particl order < B< B' => ABC<AB'C HAB,CER

- The ideal I is gan. by atts
of the form Wd - fa where
Wd TI a pronomial and for D
a lim. work of monomonials
Smaller than Wd M < .
So Wd = for in P/I and more generally
A Wd C = Afd C HA = C C R.
We can think of the out a reduction role.
Then (roughly) If reductions can be done
with no ambiguity. then the minimal/irr.
monomials in
$$\langle x \rangle$$
 form a bass of R/I.

-
$$I = \langle x & g - y & x - [x, y] \rangle$$
.
may assume we only take pairs x.y
where $X = y$, s.
 $X \otimes y - y \otimes x - [x, y]$
 $= W_d - f_d$
 $W[W_d = x \otimes y. f_d = y \otimes x - [x, y]$.
 $x \otimes y = x \otimes y. f_d = y \otimes x - [x, y]$.

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