

MATH 2001. MORE PROOF PROBLEMS

Prove the following statements.

(1) Suppose $x \in \mathbb{R}$. If $x^3 - x > 0$ then $x > -1$.

(2) Suppose $x \in \mathbb{R}$. If $x^5 - 4x^4 + 3x^3 - x^2 + 3x - 4 \geq 0$, then $x \geq 0$.

(3) Suppose $a \in \mathbb{Z}$. If a^2 is not divisible by 4, then a is odd.

(4) If $n \in \mathbb{Z}$, then $4 \nmid (n^2 - 3)$.

(5) The number $\sqrt{3}$ is irrational.

(6) Suppose $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even.

(7) Suppose $a, b \in \mathbb{R}$. If a is rational and ab is irrational, then b is irrational.

(8) There exist no integers a, b for which $21a + 30b = 1$.

(9) If A and B are sets, then $A \cap (B - A) = \emptyset$.