Math 2001. Counting Problems, Part 2
(the inclusion-exclusion principle, and problems involving multisets)

(1) State the inclusion-exclusion principle for two sets (the one computing the cardinality of $A \cup B$ for sets A, B).

(2) At a certain university 523 of the seniors are history majors or math majors (or both). There are 100 senior math majors, and 33 seniors are majoring in both history and math. How many seniors are majoring in history?

(3) State the inclusion-exclusion principle for three sets (the one computing the cardinality of $A \cup B \cup C$ for sets A, B, C).

(4) How many 7-digit binary strings begin in 1 or end in 1 or have exactly four 1's?

(5) How many 10-element multisets can be made from the symbols 1, 2, 3, 4?

(1) A bag contains 20 identical red balls, 20 identical blue balls and 20 identical green balls. You reach in and grab 15 balls. How many different outcomes are possible?

(2) A bag contains 20 identical red balls, 20 identical blue balls, 20 identical green balls, and one white ball. You reach in and grab 15 balls. How many different outcomes are possible?

(3) How many length-6 lists can be made from the symbols a, b, c, d, e, f, and g, if repetition is allowed and the list is in alphabetical order? (Examples: bbcegg, but not bbbagg.)

(4) How many lists (x, y, z) of three nonnegative integers are there with $x + y + z = 100$?

(5) How many lists (x, y, z) of three nonnegative integers are there with $x \leq y \leq z \leq 100$?

(6) How many sets $\{x, y, z\}$ of three nonnegative integers are there with $x < y < z < 100$?

(7) How many permutations are there of the letters in the word "PEPPERMINT"?