Math 2001. Lecture 8.

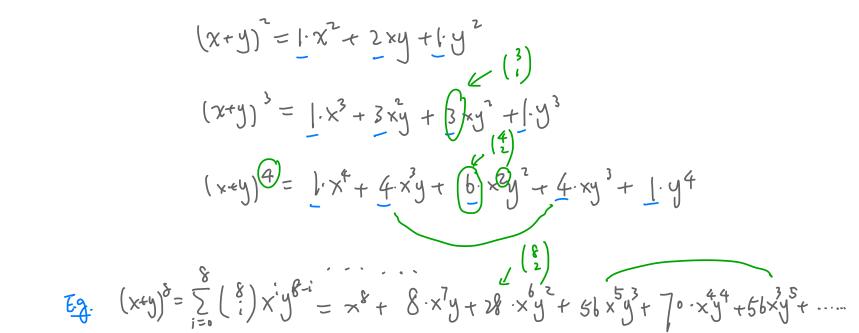
Last time: . Combinatorial proufs

· Courting worksheet

· Michterm.

Thm:
$$\forall x \in \mathbb{Z}_{\geq 1}$$
, we have $(x+y)^n = \sum_{i=0}^n \binom{n}{i} x^i y^{n-i} = \sum_{i=0}^n \binom{n}{i} x^i y^{n-i}$

Eq. $(x+y)^n = \int_{-\infty}^n (x+y)^n = \sum_{i=0}^n \binom{n}{i} x^i y^{n-i}$



Pf: One way to expand $(x+y)^n = (x+y)(x+y) - \cdot \cdot (x+y)$ is to sum all the 2" summands of the form x'y" which each results from multiplying n terms, with one from each pair of parentheses and ; of them being x. (e.g. (x+y)= (x+y) (x+y) = x.x + x.y + y.x + y.y = x.x + (2)xy + y.y. (x+y) (x+y) (x+y) = x.x.x + x.x.y + x.y.x + x.y.y + y.x.y + y.y.x+ y.y-y, = --··+3·x²y + ···) It follows that the coefficient of xiyn-i in the expansion of (xiy)h is (i), corresponding to the possibilities of picking i pairs from

Pascals Triangle: (u,i) In light of the binomial thm, the numbers the form (i) we also often cauch binomial crefficients. Binomial coefficients can be arranged into the so-called fascal's Triangle. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} z \\ \delta \end{pmatrix} \qquad \begin{pmatrix} z \\ i \end{pmatrix} \qquad \begin{pmatrix} z \\ z \end{pmatrix}$ $\begin{pmatrix} 3 \\ 3 \end{pmatrix} \qquad \begin{pmatrix} 3 \\ 3 \end{pmatrix} \qquad \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ $\begin{pmatrix} 4 \\ 6 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

Note: Each interior of in a row is the sum of the number of its two shoulders, because $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ whenever ock < n.

This allows us to compute Pascal's Triangle inductively (reconsidery)
by doing addition, without computing bir. coeff, as quotients.