MATH 4140. HOMEWORK 9

due Wednesday, April 14

Note: All numbered sections, exercises, theorems and definitions are from Erdmann–Holm.

Let k be a field.

(1) Read Chapter 5.

(2) Show that for any k-algebras $A_1, A_2, \ldots, A_r$, we have

$$(A_1 \times A_2 \times \cdots \times A_r)^{\mathrm{op}} \cong A_1^{\mathrm{op}} \times A_2^{\mathrm{op}} \times \cdots \times A_r^{\mathrm{op}}$$

as algebras.

(3) Prove that for any division algebra D over k, the transpose map

$$M_n(D^{\mathrm{op}}) \to (M_n(D))^{\mathrm{op}}, \quad X \mapsto X^T$$

is an algebra isomorphism.

(4) In the proof of Lemma 5.6, the authors show that the map Φ is bijective by checking injectivity and surjectivity separately. In class, we discussed the surjectivity proof. Complete either one of the following problems to finish the proof that Φ is bijective:

(a) Show that Φ is injective.

(b) Show that Φ is bijective by finding a two-sided inverse map.

Please use the same notation as the book/our notes. (You don't need to recall the meaning of the notation.)

(5) Exercise 5.1.

(6) Exercise 5.4.

(7) Exercise 5.9.