MATH 4140. HOMEWORK 7
due Wednesday, March 24

Note: All numbered sections, exercises, theorems and definitions are from Erdmann–Holm.

(1) Read Sections 3.4.1, 3.5 and 4.1.

(2) Exercise 2.23.

(3) Exercise 3.9.(a). If you'd like to read/review more about polynomial rings, read Sections 8.3, 9.2 and 9.4 of the book *Abstract Algebra* by Dummit and Foote. I've put the relevant excerpt in Canvas, under `Files`.

(4) Exercise 3.12.(a)–(b).

(5) Let A be a k-algebra and let V be an A-module. Prove the following statements, which are often interpreted as "quotients and homomorphic images of modules are the same things":
   (a) Let $\phi : V \to W$ be an A-module homomorphism, then $\mathrm{im}(\phi)$ is isomorphic, as an A-module, to a quotient module of V.
   (b) Let U be a submodule of V. Then there is an A-module W and a module homomorphism $\varphi : V \to W$ such that V/U is isomorphic, as an A-module, to $\mathrm{im}(\varphi)$.