MATH 4140. HOMEWORK 4

Due Wednesday, February 18

Note: All numbered sections, exercises, theorems and definitions are from Erdmann–Holm.

(1) Read Section 2.2-2.4.

(2) Assuming the isomorphism theorems for groups, prove the isomorphism theorems for modules, i.e., the statements of Theorem 2.24.

(3) Let R be a ring, let M be an R-module, and let $(U_i)_{i\in I}$ be a family of R-submodules of M. Let N be the external direct sum of $(U_i)_{i\in I}$ defined in Part (b) of Definition 2.17, and suppose that M equals the internal direct sum of $(U_i)_{i\in I}$ in the sense that it satisfies the conditions in Part (b) of Definition 2.15. Prove that the map $\phi : N \to M$ defined by $(u_i)_{i\in I} \mapsto \sum_{i\in I} u_i$ is an isomorphism of R-modules.

(4) Exercise 2.5.

(5) Exercise 2.6.

(6) Exercise 2.15.

(7) Exercise 2.22.