MATH 4140. HOMEWORK 1
Due Friday, January 29

Note: All numbered exercises are from Erdmann–Holm ([EH]).

(1) Read Sections 1.1 and 1.2 of [EH].

(2) Exercise 1.3, but change the problem to "Show that B is a K-subalgebra of A if and only if $1_A \in B$ and B itself is a K-algebra with the operations induced from A".

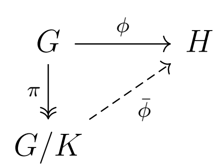
(3) Exercise 1.4.

(4) Exercise 1.8.

(5) Exercise 1.9.

(6) Exercise 1.15.(i)–(iii).

(7) (Universal Property of Quotient Groups) Let G be a group, let K be a normal subgroup of G, and let $\pi : G \twoheadrightarrow G/K, g \mapsto gK$ be the natural projection from G to G/K. Prove that for any group H and any group homomorphism $\phi : G \to H$ such that $K \subseteq \ker \phi$, there exists a unique (well-defined) group homomorphism $\bar{\phi} : G/K \to H$ such that the following diagram commutes (i.e., such that $\phi = \bar{\phi} \circ \pi$).



(8) State the First Isomorphism Theorem for groups, then use the previous exercise to prove it. (*Hint*: Start with a group homomorphism ϕ from a group G, take $H = \operatorname{im} \phi$, and take $K = \ker \phi$.)