MATH 3140. HOMEWORK 5

due Wednesday, Oct. 2

Note: As usual, justify all your answers.

(1) Find a group G, a subgroup H of G and an element g of G such that the coset gH is not a subgroup of G.

(2) Let G be a group and let H, K be subgroups of G such that $\gcd(|H|, |K|) = 1$. Prove that $H \cap K = \{1_G\}$.

(3) Let G be a group of order 4. Prove that either G is generated by a single element or $g^2 = 1$ for all $g \in G$.

(4) When you divide 31^{2019} by 12, what is the remainder?

(5) Consider the elements $a = (12)(345)$ and $b = (13)(456)$ in S_6.
   (a) Find an element $g \in S_6$ such that $b = gag^{-1}$.
   (b) Find the size of the conjugacy class of a in S_6.

(6) Describe the conjugacy classes of S_5, then compute the size of each class.

(7) Consider the dihedral group D_5. Recall that each element of the group can be written in the standard form, either as $a_i := r^i$ for some $0 \leq i \leq 4$ or as $b_i := sr^i$ for some $0 \leq i \leq 4$ (see Problem 1 of Homework 3).
   (a) Let $0 \leq i, j \leq 4$. Compute the standard form of $a_i a_j a_i^{-1}$, $a_i b_j a_i^{-1}$, $b_i a_j b_i^{-1}$ and $b_i b_j b_i^{-1}$.
   (b) Compute the conjugacy classes of D_5.

(8) Prove that the center of a group G is always a normal subgroup of G.

(9) Find all subgroups of the symmetric group S_3, then determine which of the subgroups are normal.