# Math 3140. Homework 4

## due Wednesday, Sept. 25

**Note: Justify all your answers. For some problems in this homework, you may want to use the following fact. (You don't have to prove it.)**

**Proposition.** *For any positive integers $m, n$, there exists integers $a, b$ such that $am + bn = \gcd(m, n)$, the greatest common divisor of $m$ and $n$.*

(1) Find all elements of order 2 in $S_4$.

(2) Find all elements of order 2 in the dihedral group $D_4$.

(3) Find all elements of order 2 in the dihedral group $D_n$ where $n \in \mathbb{Z}_{\geq 3}$.

(4) Find all generators of the cyclic group $\mathbb{Z}/8\mathbb{Z}$.

(5) Let $G$ be a group and let $a \in G$. Suppose $a$ is of finite order $k$ for some $k \in \mathbb{Z}_{>0}$. Show that for any $n \in \mathbb{Z}$, we have $\langle a^n \rangle = \langle a^{\gcd(k,n)} \rangle$.
(Hint: to show the two sets are equal, show that they contain each other. One direction is easy; use Proposition 1 for the other one.)

(6) Let $m, n \in \mathbb{Z}$. Find a generator for the subgroup $\langle m \rangle \cap \langle n \rangle$ of $(\mathbb{Z}, +)$.

(7) Recall that we claimed in class that for any $n \in \mathbb{Z}_{>1}$, the set $U_n := \{1 \leq k \leq n : \gcd(k, n) = 1\}$ forms a group under multiplication modulo $n$. Prove this claim.