MATH 3140. HOMEWORK 2

due Wednesday, Sept. 11

Let n be a positive integer throughout the problem set.

(1) Consider the subset $G = \{\sigma \in S_n : \sigma(1) = 1\}$ of S_n.
   (a) Prove, by checking the group axioms, that G forms a group under composition.
   (b) Find the order of G (your answer should involve n).

(2) Now assume $n \geq 4$, let $A = \{1, 2, 4\}$, and let
$$G = \{\sigma \in S_n : \sigma(a) \in A \text{ for all } a \in A\}.$$
   (a) Prove that G forms a group under composition.
   (b) Find the order of G.

(3) For each element of S_5 below, first write it in the disjoint cycle notation, then write it as a product of basic transpositions.
$$\sigma_0 = \underline{12345}, \quad \sigma_1 = \underline{13245}, \quad \sigma_2 = \underline{35412}, \quad \sigma_4 = (132)(1435).$$

(4) Let $\sigma = (123)(145) \in S_5$. Write σ^{99} in the disjoint cycle notation.

(5) Prove that S_n is abelian if and only if $n < 3$.

(6) Prove that the set $\{(1k) : 2 \leq k \leq n\}$ generates S_n. Feel free to invoke any results we have proved in class, but if you do, point out clearly what results you are using.

(7) Recall from class that the order of a permutation $\sigma \in S_n$ is given by the least common multiple of the sizes of the cycles in its cycle decomposition. Find the largest order an element of S_5 can have, and find an element with that order.