

MATH 3140. HOMEWORK 12

due Wednesday, Dec. 4.

- (1) Prove that there is no simple group of order 150.
- (2) (a) Prove that if G is a group and H is a subgroup of G of index 2, then H is normal in G .
(b) Prove that there is no simple group of order 54.
- (3) Let $G = S_6$ and let $H = \langle (123), (456) \rangle \leq G$.
(a) Show that $H \cong \mathbb{Z}_3 \times \mathbb{Z}_3$.
(b) Prove that all 3-Sylow subgroups of G are isomorphic to $\mathbb{Z}_3 \times \mathbb{Z}_3$.
- (4) Find the invariant factor decomposition of the group $\mathbb{Z}_{44} \times \mathbb{Z}_{45} \times \mathbb{Z}_{48}$.
- (5) Classify all finite abelian groups of order 1800 in terms of their primary decomposition, then find the corresponding invariant factor decompositions of these groups.
- (6) Find the invariant factor decompositions of the abelian group G generated by four elements a, b, c, d subject to the relations
$$\begin{aligned} 2a - b - c &= 0, & -a + 4b - c - 2d &= 0, \\ -a - b + 3c - d &= 0, & 2b + c - 3d &= 0. \end{aligned}$$