Math 2135. Lecture 31.

11.05.2024.

Last time:

- using coordinate mappings.

Today:

- HW 10

4.2: 2, 4, 8, 10, 14;

4.4: 2, 8, 12, 14, 28, 32;

4.5: 12, 14.

- proof and review worksheet

4.2. 2, 4, 8, 10, 14.

2. Is $w = \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}$ in Null A for $A = \begin{bmatrix} 5 & 21 & 19 \\ & * & \end{bmatrix}$?

"Is $Aw = 0$?" ← use the def of Null A.

4. Explicitly describe Null A for $A = \begin{bmatrix} 1 & -6 & 4 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$.

$\text{Null}\,A = \{ \vec{x} \in \mathbb{R}^4 \mid A\vec{x} = 0 \}$ ← solving $A\vec{x} = 0$. give the geometric description.

8. Is $S = \left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} : 5r - 1 = s + 2t \right\} \subseteq \mathbb{R}^3$ a Subspace of $\mathbb{R}^3$?

check the defining properties.

10. Is $S' = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{matrix} a+3b=c \\ b+c+a=d \end{matrix} \right\} \subseteq \mathbb{R}^4$ a subspace of $\mathbb{R}^4$?

Possible ways to approach this : (1). Check the defining properties.

(2) $S' = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{matrix} a+3b+c=0 \\ a+b+c-d=0 \end{matrix} \right\}$

$= \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{bmatrix} 1 & 3 & 1 & 0 \\ & \cdots & & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

$=$ Null A for $A = \cdots\cdots$.

Since null spaces of matrices are known to be subspaces --- , .

14. Is $\left\{ \begin{bmatrix} a+2b \\ a-2b \\ 3a-2b \end{bmatrix} : a,b \in \mathbb{R} \right\}$ a subspace of $\mathbb{R}^3$?

Could do:

(1) check def.

(2) think "span".

4.4. 2, 8, 12, 14, 28, 32

2. Find the vec $x \in \mathbb{R}^2$ s.t. $[x]_B = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$ w.r.t. the basis $B = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\}$

Easy: $x = 8 \cdot \begin{bmatrix} 4 \\ 5 \end{bmatrix} - 5 \cdot \begin{bmatrix} 6 \\ 7 \end{bmatrix} = \cdots$

8. Find $[x]_B$ for $x = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix} \in \mathbb{R}^3$ and the basis $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \right\}$ of $\mathbb{R}^3$.

Harder: we need to solve

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}. \quad \cdots$$

12. Similar to (8), use "$Ax = b \iff x = A^{-1}b$" when A is inv.

14, 28, 32 : all about P_n for some n.

Key : use the standard basis $B_0 = \{1, t, t^2, \dots, t^n\}$ of P_n

to turn the problems into problems about $\mathbb{R}^{n+1}$;

see Lecture 30.

e.g. 14 : find $[p(t)]_B$ for $p(t) = 3+t-6t^2$ and

$B = \{1-t^2, t-t^2, 2-2t+t^2\}$

decompose v using v_1, v_2, v_3.

$\Updownarrow$

decompose $[v]_{B_0}$ using $[v_1]_{B_0}, [v_2]_{B_0}, [v_3]_{B_0}$.

4.5 : 12, 14.

12. Find the dimension of $\text{Span}\left\{\begin{bmatrix}1\\-2\\0\end{bmatrix}, \begin{bmatrix}-3\\4\\1\end{bmatrix}, \begin{bmatrix}-8\\6\\5\end{bmatrix}, \begin{bmatrix}-3\\0\\7\end{bmatrix}\right\}$.

↓

find a basis of $\text{Col}\left(\begin{bmatrix}1 & -3 & -8 & -3\\-2 & 4 & 6 & 0\\0 & 1 & 5 & 7\end{bmatrix}\right)$, then count the basis.

14. find the dim. of Nul A and Col A for $A = \begin{bmatrix} \cdots \end{bmatrix}$.

↓

use the procedures from Ch2 to find a basis for Nul A and Col A.

or, find one of the dimensions and use the Rank-Nullity Thm.