Math 2130. Lecture 28.

03.26.2021.

Last time:

- Coordinate mappings $[\ ]_{\mathcal{B}} : V \to \mathbb{R}^n$; connecting arbitrary V to
  $v \mapsto [v]_{\mathcal{B}}$ a v.s. of the form $\mathbb{R}^n$.

- Row space and row rank. Thm: $\text{RowRank}(A) = \text{Col Rank}(A)$ for every matrix A.

Today: homework discussion

HW 10. (due Mar. 31.)

4.1: Ex6. Is $S = \{ p(t) = a + t^2 \mid a \in \mathbb{R} \}$ a subspace of $\mathbb{P}_n$ for some n?

Answer: No, not for any n. $\to$ check that S fails some subspace axiom.

Ex 8. Is $S = \{ p(t) \mid p(0) = 0 \}$ a subspace of P_n for some n?

P_n — the set of single-variable poly. of degree at most n.

Soln: Yes, is a subspace of P_n for every n.

Todo: Check that for all n, S satisfies the conditions.

Ex 12. Is $S = \left\{ \begin{bmatrix} s+3t \\ s-t \\ 2s-t \\ 4t \end{bmatrix} : s, t \in \mathbb{R} \right\}$ a subspace of $\mathbb{R}^4$?

"Yes. because it's a span." → you should make this precise

Ex 16. Is $S = \left\{ \begin{bmatrix} -a+1 \\ a-6b \\ 2b+a \end{bmatrix} : a, b \in \mathbb{R} \right\} \subseteq \mathbb{R}^3$ a subspace of $\mathbb{R}^3$?

(Is 0 in S?)

Ex 18. See Ex 12. & Ex. 16.

4.2:

10. Is $S = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{array}{l} a+3b=c \\ b+c+a=d \end{array} \right\} \subseteq \mathbb{R}^4$ a subspace of $\mathbb{R}^4$?

Strategy: realize S as a kernel of a linear map / null space of a matrix A.

$$\begin{cases} a+3b-c = 0 \\ a+b+c-d = 0 \end{cases}$$

$\longrightarrow A = \begin{bmatrix} \cdots \cdots \end{bmatrix}$

12. Similar to 10. but there's a "+1".

$\downarrow$

Is 0 in the set?

4.4. 28. Is $\{ \underset{v_1}{1-2t^2-t^3}, \underset{v_2}{t+2t^3}, \underset{v_3}{1+t-2t^2} \}$ a lin. ind. set (say in P_3)?

Use coordinate vectors w.r.t. to $B = \{1, t, t^2, t^3\}$ of P_3 $\left(v_1 \mapsto \begin{bmatrix} 1 \\ 0 \\ -2 \\ -1 \end{bmatrix} =: u_1, \quad \dots \right)$, thereby turning the question into whether u_1, u_2, u_3 are lin. ind. in $\mathbb{R}^4$.

4.6. 2 & 4. Given A and $EF(A)$, find bases for $\mathrm{Col}\,A$, $\mathrm{Row}\,A$, and $\mathrm{Nul}\,A$.

$\rightarrow$ Consult the corresponding algorithms.

6 & 8. Use the rank-nullity theorem:

$$\dim(\mathrm{Nul}\,A) + \underset{\substack{\parallel \\ (\dim(\mathrm{Col}\,A))}}{\mathrm{rank}\,A} = \#\text{ cols of } A.$$

and use the fact that $\mathrm{Col\,Rank}\,A = \mathrm{Row\,Rank}\,A \overset{\substack{\text{def} \\ \text{of} \\ A^T}}{=\!=} \mathrm{Col\,Rank}\,A^T$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 5 \end{bmatrix}$$