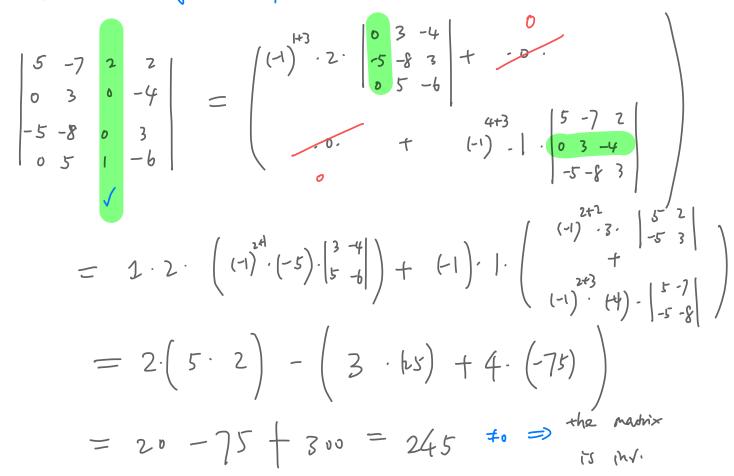
Math 2130. Leiture 24.

Example. Determinant of a 4x4 matrix.



A shortcut (or not) for computing 3x3 det: aei + bfg + cdh - ceg - bdi - ahf.

1. Properties of Determinants
Let A be an num module. (eq.
$$\begin{bmatrix} x & y \\ z & w \end{bmatrix}$$
 or $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$)
Facts. (Effect of yow operations on determinants)
i) (Effect of scaling) Fact: If we scale one now of a square module
A to obtain a square module B by a scalar C, then
det B = c det A
Eq. A = $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ $\xrightarrow{R_1 \to c.R_2}$ B = $\begin{pmatrix} x & y \\ cz & cw \end{bmatrix}$: det B = $x(cw) - y \cdot (cz)$
 $= c(xw - yz) = c det A$.
A : $2xz = 2$ det $(-A) = (-1)^2$ -det A = det A; A: non = 2 det $(-A) = (-1)^2 det A$.

(7). Effect of interchange
Funt: If we swap two rows in a square matrix
$$A$$
 to obtain a matrix B ,
then $\det B = -\det A$.
eg. $\det \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = 1.4 - 2.3$ $\det \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = 3.2 - 1.4 = -\det B$
Ex: Convince yourself what $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -\begin{bmatrix} a & b & c \\ 3 & h & i \\ d & e & f \end{bmatrix}$.
Corollary: If a square matrix has a row which is the same of another row.
then it has det. 0.
Pf: $\det A = -\det (A, with Row i, Row j swapped) = -\det (A) = \det A = 0.$

Considery': If A is a square metrix where one now is a scalar multiple f
another, then det
$$A = 0$$
.
Pf. Ex. eg. $\begin{vmatrix} a & b & c \\ d & e & f \\ xa & xb & xc \end{vmatrix} = X \begin{vmatrix} a & b & c \\ d & e & f \\ a & b & c \end{vmatrix} = x \cdot 0 = 0$.
3) (Effect of replacement) Fact: If we add a multiple of a new to a sq.
matrix to another now, then the resulting matrix has the same det as
the original matrix. (det $B = det A$).
 $\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = \begin{vmatrix} 7 & 7 \\ 1 & 0 \end{vmatrix} = \begin{vmatrix} 7 & 7 \\ 1 & 0 \end{vmatrix}$
eff. $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} A & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} A & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} 7 & 7 \\ 1 & 0 \end{vmatrix}$
A relaxed fast: $\begin{vmatrix} \frac{Y_{1}}{Y_{2}} \\ \frac{Y_{1}}{Y_{1}} \end{vmatrix} + \begin{vmatrix} \frac{Y_{1}}{Y_{2}} \\ \frac{Y_{1}}{Y_{1}} \end{vmatrix} = \begin{vmatrix} \frac{T_{1}+T_{1}'}{Y_{2}} \\ \frac{Y_{1}}{Y_{1}} \end{vmatrix}$

Note: If two square matrices A.B are now equivalent (i.e., B can
be obtained from A via a sequence of ext now operations), then det A and
det B are either both D or both nonzero.
An application of the properties: We can now prove Thm 1:
Thm 1: Let A be a square matrix. Then
det A #D
$$\Leftrightarrow$$
 A 3 inv.
Pf: A 73 inv \Leftarrow ? REF (A) = In \Leftarrow ? det (REF(A)) = 0
But A is now equive to REF(A), so det (REF(A)) = $(e^{-1}A^{-1})^{-1}A^{-1} + (e^{-1}A^{-1})^{-1}A^{-1} + (e^{-1}A^{-1})^{-1}A^$