Math 2001. Proof Problems, Part 4
(Mathematical Induction)

Prove the following statements.

(1) In the Fibonacci sequence, we have $F_1 + F_2 + \cdots + F_n = F_{n+2} - 1$.

(2) In the Fibonacci sequence, we have $\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$.

(3) The n-th Fibonnaci number F_n is even if and only if $3 \mid n$.

(4) We have $\sum_{i=1}^{n} i^3 = \dfrac{n^2(n+1)^2}{4}$ for every positive integer n.

(5) Suppose $A_1, A_2, \ldots, A_n$ are sets in some universal set U, and suppose $n \geq 2$. Then

$$\overline{A_1 \cap A_2 \cap \cdots \cap A_n} = \overline{A}_1 \cup \overline{A}_2 \cup \cdots \cup \overline{A}_n.$$

(6) If $n \in \mathbb{Z}_{>0}$, then $1/1 + 1/2 + 1/3 + \cdots + 1/(2^n - 1) + 1/2^n \geq 1 + n/2$.