

MATH 2001. PROOF PROBLEMS, PART 3
(Combined techniques)

Each of the following statements is either true or false. If a statement is true, prove it; otherwise, disprove it.

(1) If $n \in \mathbb{Z}$ and $n^5 - n$ is even, then n is even.

(2) If A, B, C are sets such that $A \times C = B \times C$, then $A = B$.

(3) If A, B, C are sets, then $A - (B \cup C) = (A - B) \cup (A - C)$.

(4) If A and B are finite sets, then $|A \cup B| = |A| + |B|$.

(5) If A and B are sets, then $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$.

(6) There exist prime numbers p and q for which $p - q = 97$.