Math 2001. Proof Problems
(direct proofs of conditional statments)

Prove the following statements.

(1) (Ex.4.4) Suppose $x, y \in \mathbb{Z}$. If x and y are odd, then xy is odd.

(2) (Ex.4.10) Suppose $a, b \in \mathbb{Z}$. If $a \mid b$, then $a \mid (3b^3 - b^2 + 5b)$.

(3) (Ex.4.13) Suppose $x, y \in \mathbb{R}$. If $x^2 + 5y = y^2 + 5x$, then $x = y$ or $x + y = 5$.

(4) (Ex.4.16) If two integers have the same parity, then their sum is even.

(5) (Ex.4.20) If a is an integer and $a^2 \mid a$, then $a \in \{-1, 0, 1\}$.

(6) (Ex.4.26) Every odd integer is a different of two squares. (For example, $7 = 4^2 - 3^2$.)