Math 2001. Lecture 33.

04. 15. 2022.

Last time:

- More properties of Fibonacci numbers
- Worksheet on inductive proofs

Today: hints for the worksheet problems

Next time: new chapter. Ch 11. Relations

P1.

Base case: easy. need $F_1 = F_3 - 1$. ie., $1 = 2 - 1$. ✓

Inductive step:
$$F_1 + \cdots + F_n = F_{n+2} - 1$$
$$+F_{n+1} \qquad +F_{n+1}$$
$$\Downarrow$$
$$F_1 + \cdots + F_n + F_{n+1} = F_{n+3} - 1$$

P2.

Base case: easy.

Inductive step:
$$\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$$
$$+F_{n+1}^2 \downarrow \qquad \Downarrow \qquad \downarrow +F_{n+1}^2$$
$$\sum_{i=1}^{n+1} F_i^2 = F_{n+1} F_{n+2}$$

P3.

F_1	F_2	F_3	F_4	F_5	F_6	F_7	$\cdots\cdots$
1	1	2	3	5	8	13	

S_n

$3 \mid n$: even

$3 \nmid n$: odd.

Base cases: $n = 1, 2, 3$. ✓

Inductive step: Suppose S_n holds for all $n \le k$ for some $k \ge 3$. We want to show S_n holds for $n = k+1$. We have

$$S_n = S_{n-1} + S_{n-2}$$

Three cases depending on the remainder of n mod 3.

Case (1) if $3 \mid n$. then $3 \nmid n-1$, $3 \nmid n-2$. so by the strong inductive hyp. F_{n-1}, F_{n-2} are both odd. $\Rightarrow F_n$ is even.

Cases (2) and (3) are similar.

P4.

Similar to $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

Interesting fact to note: summing over pth order expressions in i gives an $(p+1)$th order expression in n.

P5.

Base case: DeMorgan's Law

Key to the inductive step: $\overline{A_1 \cap A_2 \cap \cdots \cap A_n \cap A_{n+1}}$

$$= \overline{(\underbrace{A_1 \cap A_2 \cap \cdots \cap A_n}_{x}) \cap \underbrace{A_{n+1}}_{y}}$$

apply DeMorgan's Law on x and y and then apply the ind. hyp on $\overline{x}$.

P6.

Base case: straightforward

Inductive step: $1/1 + 1/2 + \cdots + 1/2^{n-1} + 1/2^n \geq 1 + n/2$

$1/8$

$$\Downarrow$$

$$1/1 + 1/2 + \cdots + 1/2^n + 1/(2^n+1) + \cdots + 1/(2^{n+1}-1) + 1/2^{n+1} \overset{?}{\geq} 1 + \frac{n+1}{2}.$$

$$\text{e.g. } n=3 \quad \underbrace{1/8 + 1/9 \cdots + 1/15 + 1/16}_{[2^{n+1} - (2^n+1) + 1] = 2^n \text{ summands}}$$

To show "$\overset{?}{\geq}$", use $\quad > 2^n \cdot \frac{1}{2^{n+1}} = \frac{1}{2}$

Rmk: The problem shows that the "harmonic series" $\sum_j 1/j$ diverges to ∞.