Math 2001. Lecture 27.

03. 30. 2022.

Last time:

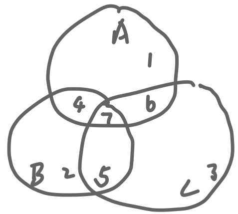
- proofs of set equalities
- disproofs of universal and conditional statements by counterexamples

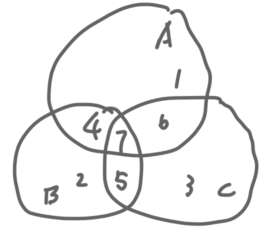
E.g. $A-(B\cap C) = (A-B)\cap(A-C)$?

Today:

- finishing the example
- disproofs of existence claims
- Worksheet 3 on proofs

1. $A-(B\cap C) = (A-B)\cap(A-C)$ (for any sets A, B, C) ?





$B\cap C$: $5, 7$

$A-(B\cap C)$: $1, 4, 6$

$A-B$: $1, 6$

$A-C$: $1, 4$

$(A-B)\cap(A-C)$: 1

$LHS = \{1,4,6\}$
$\neq$
$RHS = \{1\}$.

So, the two side ~~probably~~ not equal. In fact, they can't be equal whenever Region 4 or Region 6 is nonempty. Better: the labelling gives a natural counter example: take $A = \{1,4,6,7\}$, $B = \{2,4,5,7\}$, $C = \{3,5,6,7\}$. $\square$

2. Disproofs of existence claims

Claim: There is some object (in an ambient universe) satisfying certain properties.

E.g.: "There is a real number x (in $\mathbb{R}$) s.t. $x^4 < x < x^2$." (*)

Rmk: To disprove such a claim, i.e., to show that no object (in the ambient universe) satisfies the given properties. It is not enough to pick a random object from the universe and check that that particular object doesn't satisfy the condition.

E.g. If $x = 2$, then it's not the case that $16 < 2 < 4$. but this example alone doesn't disprove (*).

One disproof of (*) "there is a number x s.t. $x^4 < x < x^2$".

Rephrase: It's impossible for a real number $x \in R$ to satisfy $x^4 < x < x^2$,

ie, to satisfy both $x^4 < x$ and $x < x^2$.

Suppose a number $x \in \mathbb{R}$ satisfies $\underline{x^4 < x < x^2}$.

Since $\underline{x < x^2}$, $x^2 - x > 0$, $x(x-1) > 0$. so either $\begin{cases} x > 0 \\ x-1 > 0 \end{cases}$ or $\begin{cases} x < 0 \\ x-1 < 0 \end{cases}$,

ie. either $\begin{cases} x > 0 \\ x > 1 \end{cases}$ or $\begin{cases} x < 0 \\ x < 1 \end{cases}$, ie, either $\underset{①}{\underline{x > 1}}$ or $\underset{②}{\underline{x < 0}}$.

① If $x > 1$, then $x^3 > 1$, so $x^4 > x$.

② If $x < 0$, then $x^4 > 0 > x$.

It follows that no $x \in \mathbb{R}$ can satisfy $x^4 < x < x^2$

3. Worksheet.