Last time: · proof by contradiction.

· writing advice

Today :   · one more proof by contradiction : infinitude of primes

· Worksheet 2 on proofs

1. Infinitude of prime numbers

Prop: There are infinitely many prime integers.

Pf: Suppose otherwise, i.e., suppose there are only finitely many primes. Then we can list them as $p_1, p_2, \ldots, p_k$ for some $k \in \mathbb{Z}_{>1}$ in increasing order. Consider the number $N := p_1 p_2 \cdots p_k + 1$. The number N must have a prime factor, which has to be p_i for some $1 \le i \le k$. Thus, we have $N = c p_i$ for some $c \in \mathbb{Z}$, and we have

$$c p_i = p_1 p_2 \cdots p_k + 1.$$

It follows that

$$c p_i - p_1 p_2 \cdots p_k \overset{*}{=} 1 \quad .$$

Now, since $p_i | c p_i$ and $p_i | p_1 \cdots p_k$, we have $p_i | c p_i - p_1 p_2 \cdots p_k$; but since $p_i > 1$, we have $p_i \nmid i$, so we have a contradiction to $*$.

It follows that there are infinitely many primes. $\square$