Math 2001. Lecture 17.

02. 23. 2022.

Last time:

- the division principle:

  if we place n objects into k boxes, at least one box get $\lceil \frac{n}{k} \rceil$ or more objects.

- a comb. proof of $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$

Today.

- One more comb. identity $\binom{2n}{2} = 2\binom{n}{2} + n^2$

- Topics for Midterm 1.

- Counting Worksheet 2.

1. $\binom{2n}{2} = 2\binom{n}{2} + n^2$

Pf: Consider picking 2 elts out of the union $A \cup B$ where $A = \{a_1, \dots, a_n\}$ and $B = \{b_1, \dots, b_n\}$ are sets with n elts each.

There are clearly $\binom{|A \cup B|}{2} = \binom{2n}{2}$ to do so.

On the other hand, we could pick these 2 elts in one of the following ways:

(1) Pick them both from A. $\longrightarrow$ $\binom{n}{2}$ ways to do this

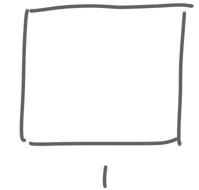
(2) . . — - . . B $\rightarrow$ $\binom{n}{2}$ - - - - . . . .

(3) Pick one from A and the other from B (the only other option)

$\longrightarrow$ $\binom{n}{1} \cdot \binom{n}{1} = n^2$ ways to do this.

It follows that $\binom{2n}{2} = 2\binom{n}{2} + n^2$.

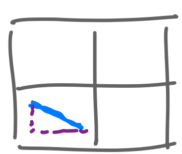
Hw. Q: 3.9.4. Place 5 points into the square



Show that two of the points must have a distance of $\frac{\sqrt{2}}{2}$ or less.

Hint: We have the objects: 5 points

need boxes: ideally any two points in a box has distance $\frac{\sqrt{2}}{2}$ or less.



(4 equal-size small squares) works.

Math 2001. Topics for Midterm 1

You should master the following topics for Midterm 1.

(1) Notation for sets, including the set-builder notation.

(2) The difference between the notations $\in$ and $\subseteq$.

(3) Definition of Cartesian products and power sets of sets, and how to count them.

(4) Definition of complements of sets.

(5) Visualizing sets (including unions, intersections and complements) using Venn diagrams.

(6) Notation for indexed sets.

(7) Truth tables for "and", "or", "not", conditional, and biconditional statements.

(8) DeMorgan's Law.

(9) Using truth tables to prove two statements are or are not logically equivalent.

(10) Notation for quantifiers.

(11) The multiplication, addition and subtraction principles for counting.

(12) Counting problems involving permutations and combinations, including problems requiring the bars-and-stars method and the word problem (see the two worksheets on counting.)

(13) The binomial theorem and its applications.

(14) Statements and applications of the inclusion-exclusion principle for two or three sets.

(15) Statements and applications of the pigeonhole and division principles.