Math 2001. Introduction to Discrete Mathematics

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Course Information:

Instructor: Tianyuan Xu (Eddy) Math 202.

Topics: Basic set/logic theory, enumeration techniques, useful/typical proof methods

Website: https://math.colorado.edu/~tixu6187/2001.html

— Lecture summaries, notes, hw and other materials will be posted under "LECTURES"

— has the Canvas link too.

Office Hours: By appointment.

Grading: HW 20%, Midterms 20% × 2, Final Exam 40%

HW:
- Posted on the website every Wed, due at 11:59 pm the next Wed.
- the deadline is strict.
- to be submitted in pdf form at Canvas/Assignment

Textbook: Book of Proof, by Richard Hammack

(in Canvas/Files and on the course website)

Today:

1. Basic Notions (for sets)

Def:

- (set & element) A <u>set</u> is a collection of objects. The objects are called the elements of the set.

- (finiteness) A set is <u>finite</u> if it contains a finite number of (distinct) elements, and is <u>infinite</u> otherwise.

- (Cardinality) The number of elts (elements) in a set A is the <u>cardinality</u> of the set. We often denote the cardinality by $|A|$.

Notation: We often write a set in one of two forms:

(1) list all the elts and enclose them in a pair of braces $\{\quad\}$,

e.g. $\{1,2,3,4\}$ is a finite set of cardinality 4.

(2) in the form $\{\text{expression} : \text{rules}\}$ or $\{\text{expression} \mid \text{rules}\}$.

(the set-builder notation)

the elts

e.g. $\{2n \mid n \text{ is an integer}\}$ is the set of all even integers.

E.g. (The empty set) We allow the empty set $\{\ \}$, a set containing no elts.

E.g. • $\{n \in \mathbb{Z} : 0 < n < 5\} = \{1,2,3,4\}$,

• We will use $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$ to stand for the sets of all integers, rational numbers and real numbers, respectively.

- $\{x \in \mathbb{R} : x^2 = 2\} = \{\sqrt{2}, -\sqrt{2}\}$

- $\{n \in \mathbb{Z} : |n| < 3\} = \{-2, -1, 0, 1, 2\}$

- Recall that $\mathbb{Q} = \left\{ x \in \mathbb{R} \;\middle|\; x = \frac{m}{n} \text{ for some } m, n \in \mathbb{Z} \text{ and } n \neq 0 \right\}$.

One more notation :

We'll denote the empty set by $\emptyset$.

2. Subsets and equality of sets

Def: A <u>subset</u> of a set A is a set B consisting (only) of objects from A.

i.e., a set B s.t. every elt of B is an elt of A.

Two different notations:

- "elt in set" We write things like "$a \in A$" to mean that a is an elt of A. e.g. $1 \in \{1,2,3,4\}$, $3 \in \{1,2,3,4\}$.

- "set contained in set" We write $B \subseteq A$ to indicate that B is a subset of A, e.g. $\{1\} \subseteq \{1,2,3,4\}$. $\{2,3\} \subseteq \{1,2,3,4\}$

E.g. $3 \in A := \{3, 5, \{-2, 3\}\}$, $\{5\} \notin A$. $\{-2,3\} \in A$. $\{3, 5\} \subseteq A$.

Def: We say two sets A, B are <u>equal</u> if they contain the same elts.

Equivalently, A, B are equal if every elt of A is in B and every elt of B is in A.

Equivalently, A, B are equal if $A \subseteq B$ and $B \subseteq A$.

E.g. $\{n \in \mathbb{Z} \mid |n| < 3\} = \{-2, -1, 0, 1, 2\}$.

E.g. A nontrivial equality. Let $A = \{2a + 5b \mid a, b \in \mathbb{Z}\}$.

e.g. $a = 1, b = 1, \quad 2a + 5b = 7$

$a = 1, b = 2, \quad \cdots \quad 12,$

$a = -3, b = 1, \quad \cdots \quad = -1.$

Proposition: $A = \mathbb{Z}$.

Analysis: The prop. claims an equality of two sets A and $\mathbb{Z}$. We can prove it by proving $A \subseteq \mathbb{Z}$ and $\mathbb{Z} \subseteq A$. To prove "$X \subseteq Y$", we need to show every elt of X is in Y.

Pf: We prove $A = \mathbb{Z}$ by proving $A \subseteq \mathbb{Z}$ and $\mathbb{Z} \subseteq A$.

(1). $A \subseteq \mathbb{Z}$: $A = \{2a + 5b : a, b \in \mathbb{Z}\}$.

Every elt in A is of the form $x = 2a + 5b$ for some $a, b \in \mathbb{Z}$, which is still an integer, i.e., $x \in A$. So $A \subseteq \mathbb{Z}$.

(2) $\mathbb{Z} \subseteq A$: next time. (Hint: show $1 \in A$ first.)