Math 4140: Homework 11

Due: April 13, 2011

Required

1. The character of a representation is obtained by composing the representation with the trace map. This problem examines what happens if you instead compose with the determinant map.

   (a) Show that if $\rho : G \to \mathrm{GL}_n(\mathbb{C})$ is a representation of G, then $\rho \circ \det : G \to \mathbb{C}$ is a character.

   (b) If ρ_V is the permutation representation of S_n, what irreducible character of S_n is

   $$\rho_V \circ \det?$$

2. (a) The character table of S_5 is given on page 201 of the textbook. However, the rows are not labeled by partitions (as we know they should). Compute enough values of the character table of S_5 so that you can identify each row with a partition of 5. For example, since χ_1 is clearly the trivial character, the corresponding shape will be (5) (because $S_5^{(5)}$ is the trivial module).

   (b) The character $\chi^{(3,2)}$ corresponding to the S_5-module

   $$S_5^{(3,2)}$$

   is irreducible. Note that since $S_4 \subseteq S_5$, this same module is also a module for S_4 (though not necessarily irreducible). Thus, as a character of S_4, $\chi^{(3,2)}$ can be written as a linear combination of irreducible characters of S_4. Use the character table we constructed in class to explicitly write down this decomposition.

3. Let G be a group, and let K and L be conjugacy classes of G. Show that using the usual inner product on $C(G)$ that

   $$\langle \kappa_K, \kappa_L \rangle = \begin{cases} \frac{1}{C_G(g_K)}, & \text{if } K = L, \\ 0, & \text{otherwise,} \end{cases}$$

   where g_K is some element in the conjugacy class K. That is, the characteristic class functions are orthogonal but not orthonormal.

Recommended

Chapter 14. 1, 2, 5