Imagine an experiment where you roll two fair dice, and observe the number appearing on each.

1. Using just your intuition, answer this question: *given* that one of the dice lands on 6, what's the probability that the other die also lands on 6?

Answer: P(one die lands on 6 given that the other does)= $1/6 \approx 0.1667 = 16.67\%$ (express as a fraction and as a percent). (How one die lands shouldn't depend on how the other does, so the probability of one landing on a six should be the same whether or not the other does.)

2. Now let's test our intuition. Do the following:

(a) Roll your two dice.

(b) If (at least) one of the dice lands on 6, then put a tally mark in the first row of the frequency table below, where it says "Event B: (at least) one die lands on 6." Otherwise, do nothing.

(c) If you *did* put a tally mark in the event B row – that is, if at least one of your dice landed on 6 – AND if the other die landed on 6 as well, then put a tally mark in the second row of the frequency table below, where it says "Event A: both dice land on 6." Otherwise, do nothing.

Now do (a,b,c) until you have *at least* 5 tally marks in the second row (the "Event A" row). (If getting five tally marks there happens quickly, feel free to keep going for a while longer.) Then, divide the number of tally marks in the second row (the "Event A" row) by the number of tally marks in the first row (the "Event B" row). What is the number you got?

Answer: $\frac{n(A)}{n(B)} =$ $63/500 = 0.126 = 12.6\%$ (express as a fraction and as a percent).

Event	Frequency
Event B: (at least) one die lands on 6.	500
Event A: both dice land on 6.	63

3. Explain why the number $n(A)/n(B)$ you got at the end of Exercise 2 above should give you some sort of idea of the probability

P(one die lands on six given that the other does).

Explanation: We are observing whether one die lands on six, and only then do we consider whether the other does as well. So it makes sense that dividing the number of double sixes by the number of outcomes with at least one six should give us an idea of the probability of two sixes given that there is at least one.

4. Does your answer from Exercise 3 above agree with your intuition from Exercise 1? Please explain.

Not really. We expected a probability of about 16.67%, but experimentally, this probability looks more like 12.6%.

5. If your answer to Exercise 4 above is "no," think about what's going on a bit. If your answer to Exercise 4 above is "yes," you should still think about it. We'll discuss this more on Monday.

See the lecture notes for September 18. There, we'll see that the *true* probability here is $1/11 \approx 0.09090 = 9.09\%$!!! Wait, what? Surprising but true.