Math 3510: Intro to Prob/Stats **Activity: Coin flipping, events, and probabilities**

Imagine an experiment where you flip six fair coins, and observe whether each coin lands heads or tails.

1. Describe the sample space S for this event. You can write down all the outcomes, but this might take a while, so you could alternatively just describe the sample space in words. Please do write down a few examples of outcomes, though, just to be concrete.

$S = \{HHHHHH, HHHHHT, HHHHTH, \ldots, TTTTTH, TTTTTT\}$. S is the set of all possible ways in which the six coins can land.

2. What is $n(S)$? That is, how many elements does S have? Please explain.

There are two ways for each of the six coins to land, so there are $2^6 = 64$ total possible outcomes.

3. Write down the event A_1 where exactly one coin land heads, by simply listing all of the outcomes in this event. Use proper set notation (I'll get you started):

$$A_1 = \left\{ \begin{matrix} HTTTTT, & THTTTT, & TTHTTT \\ TTTHTT, & TTTTHT, & TTTTTH \end{matrix} \right\}$$

4. What is $P(A_1)$? Please explain.

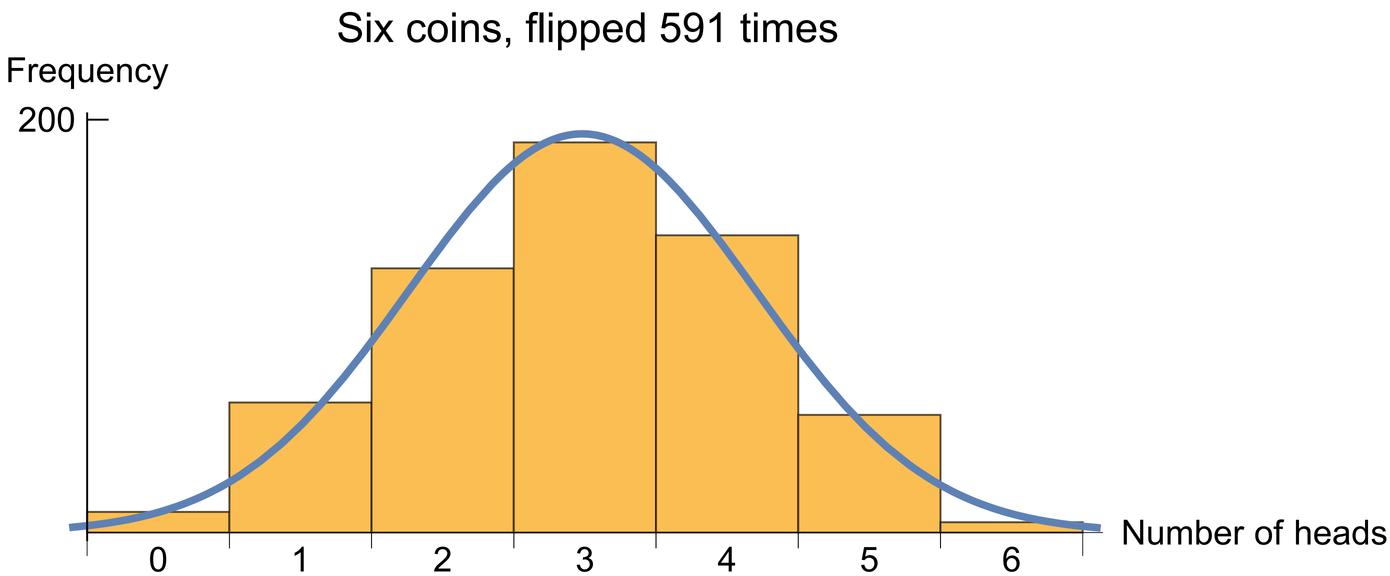
$P(A_1) = 0.09375 = 9.375\%$. This is because, in a sample space where all outcomes are equally likely, the probability of any event A equals $n(A)/n(S)$, where S is the sample space. So in this case

$$P(A_1) = n(A_1)/n(S) = 6/64 = 0.09375 = 9.375\%.$$

5. Let A_0 denote the event "0 coins land heads," let A_1 denote the event "1 coin lands heads," let A_2 denote the event "2 coins land heads," etc. Flip your six coins, and put a tally mark in the corresponding slot on the "frequency table" below. (That is: if you get three heads, put a tally mark in the "A_3" row of the "Frequency" column, and so on.) Repeat this experiment *many, many more* (at least 150!!) times, tallying your results each time in the table below. **In the table, please mark down not only your tally marks, but also the total tally (count), in each row.**

Event	Frequency
A_0	10
A_1	63
A_2	128
A_3	189
A_4	144
A_5	57
A_6	5

6. On the axes below, draw a histogram depicting the information from your above frequency table. PLEASE BE NEAT.



7. Does the number of times that exactly one coin landed heads, in your histogram, agree (roughly) with your answer to Problem 4 above? Please explain.

In our experiment, we had exactly one coin land heads 63 out of 591 times. This means that event A_1 happened with a relative frequency of $63/591 = 0.106599 \approx 10.65\%$, which is not that far off from our theoretical probability of 9.375%.

8. Suppose the experiment were to flip 50 coins, instead of 6, at a time. Suppose you were to repeat this experiment thousands of times. Describe, in general terms, what you think the histogram would look like. (About where do you think this histogram would peak? What kind of shape would it have?)

We'd expect the histogram to follow something of a normal (bell-shaped) pattern. Out of 50 possible heads, we'd expect around 25 coins to lands heads more often than any other event, with small or large numbers of heads occurring less and less often, the further we get from 25 heads. Here are some simulated results: