In this worksheet, we look at several ideas from probability: outcomes; sample space; events; probabilities of outcomes and events.

Consider the experiment where you roll two dice – say, a red die and a white die – and write down a pair of digits between 1 and 6, corresponding to the numbers that come up on the red and the white die respectively. For example, if the red die shows a 5 and the white die shows a 3, then the recorded outcome would be 53.

1. Write down the sample space S for this experiment (that is, the set of all possible outcomes). Don't just describe the sample space in words; instead, list all possible outcomes (for example, 53 is one outcome). Use proper set notation. (This sample space is relatively large. Be patient.)

$S =$

2. Let's assume that the dice are fair, so that the outcomes of your experiment are all equally likely. What **is** the probability, let's call it $p(\omega)$, of any outcome ω? Write it as a fraction. Hint: the two dice are independent of each other.

$p(\omega) =$_____.

Explain how you know this. (Refer to results from the text and/or class notes, if you wish.)

Now Let A_2 be the event where the *sum* of the numbers appearing on the dice equals two, A_3 the event where the sum equals three, A_4 the event where the sum equals four, and so on.

3. Write down each of the sets $A_2, A_3, A_4, \ldots, A_{12}$, by listing the elements in each (much as you did for the sample space S above):

$A_2 =$

$A_3 =$

$A_4 =$

$A_5 =$

$A_6 =$

$A_7 =$

$A_8 =$

$A_9 =$

$A_{10} =$

$A_{11} =$

$A_{12} =$

4. Compute $P(A_2), P(A_3), P(A_4), \ldots$, up to $P(A_{12})$. Hint: since we're in the equally likely case here,

$$\mathrm{P}(A_3) = \frac{\text{number of ways the dice can sum to three}}{\text{size of sample space}},$$

etc. As above, express your answers as fractions (you don't need to reduce them). Hint for checking your work: the probabilities you get must add up to one.

$P(A_2) =$

$P(A_3) =$

$P(A_4) =$

$P(A_5) =$

$P(A_6) =$

$P(A_7) =$

$P(A_8) =$

$P(A_9) =$

$P(A_{10}) =$

$P(A_{11}) =$

$P(A_{12}) =$

5. Suppose you were to repeat the above experiment, of rolling two dice, 6000 times, and each time you record the *sum* of the two numbers that come up. On the axes given below, draw a histogram (like the one we did on class on Friday) of what you think the results would look like, more or less. Please explain your results, in just a sentence or two (or three or four).