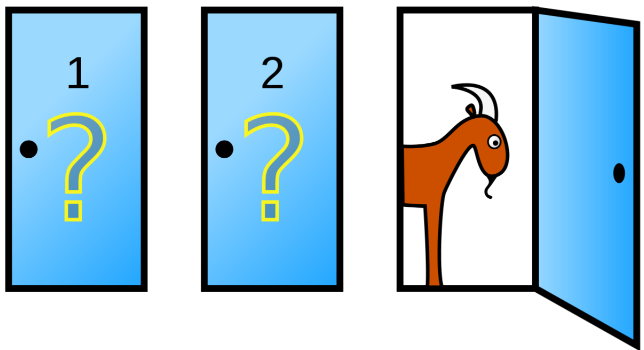
In this problem set, we consider "the Monty Hall Problem," which is a probabilistic curiosity.

Consider this hypothetical scenario from "Let's Make a Deal," a popular TV game show that ran for decades starting in the sixties, hosted by Monty Hall:

> A contestant is shown three doors, and told that there is a new car behind one of them, and a goat behind each of the other two. The contestant chooses a door. Monty Hall **does not** open the chosen door, but **does** reveal a goat behind one of the two doors the contestant did *not* choose.



> Monty Hall then asks the contestant, "Would you like to stick with your first choice, and take whatever lies behind the door you just picked, or instead switch, and go with whatever lies behind the other closed door?"

QUESTION: is the contestant better off switching, or sticking with their original choice? ("Better off" means "more likely to win the car.")

Intuitively, it seems like it shouldn't matter – the contestant has no information about what's behind either of the two closed doors. Right?

Well, let's see. To answer this question, we consider each strategy (STICKING and SWITCHING) separately, and calculate the probability of winning in each case. To this end, let's give names to the three doors: let's call the door with the car behind it W, and call the other two doors L_1 and L_2. Of course, the contestant doesn't know which door is which, though the contestant **does** know, once Monty reveals a goat behind one of the doors, that this door (the one Monty opens) is either door L_1 or door L_2.

Part A: STICKING strategy. This is the strategy where the contestant DOES NOT SWITCH after a goat is revealed behind one of the doors.

Fill in the blanks (there are three of them): Since the contestant is **not** going to switch, the contestant will win if their original pick is the ______________ (W, L_1, or L_2) door, and will lose otherwise. Since there are **three** doors, and only **one** of them is the ______________ (W, L_1, or L_2) door, the probability that the contestant will win, with this STICKING strategy, is therefore equal to ______________________ (a fraction, like 1/2, 4/7, etc.).

Now we consider

Part B: SWITCHING strategy. This is the strategy where the contestant DOES SWITCH after a goat is revealed behind one of the doors. There are three cases to consider here:

Case (i). Fill in the blank (there is just one): Suppose the door the contestant picks in the first place (before seeing the goat) is the W door. Then, since the contestant **is going to switch**, the contestant will _______________ (win, lose).

Case (ii). Fill in the blanks (there are three of them): Now suppose the door that the contestant picks in the first place is the L_1 door. Monty then reveals a goat behind one of the remaining two doors, so that door (the one Monty opens) *must* be the _______________ (W, L_1, or L_2) door. So the remaining door – the one the contestant is going to switch to – must be the _______________ (W, L_1, or L_2) door. CONCLUSION: if the first door the contestant picks is the L_1 door, then by switching, the contestant is guaranteeed to _______________ (win, lose).

Case (iii). Fill in the blanks (there are three of them): Now suppose the door that the contestant picks in the first place is the L_2 door. Monty then reveals a goat behind one of the remaining two doors, so that door (the one Monty opens) *must* be the _______________ (W, L_1, or L_2) door. So the remaining door – the one the contestant is going to switch to – must be the _______________ (W, L_1, or L_2) door. CONCLUSION: if the first door the contestant picks is the L_2 door, then by switching, the contestant is guaranteed to _______________ (win, lose).

SUMMARY of SWITCHING strategy: there are three doors total, and under the SWITCHING strategy, precisely _______________ (0, 1, 2, or 3) of these doors will yield a win if chosen in the first place. So the probability of winning, under the SWITCHING strategy, is _________________________ (a fraction, like 1/2, 4/7, etc.).

Part C: Comparison of the two strategies. Fill in the blanks (there are three of them): In **Part A** above we saw that, with the STICKING strategy, the contestant's probability of winning is _________________________ (a fraction, like 1/2, 4/7, etc.). On the other hand, in **Part B** above we saw that, with the SWITCHING strategy, the contestant's probability of winning is _________________________ (a fraction, like 1/2, 4/7, etc.). Therefore the _________________________ (STICKING, SWITCHING) strategy is the better strategy for winning.

Part D: Reflection. Looking back, can you now explain, intuitively (using as little actual math as possible), why one strategy is better than the other?