Random variables, continued.

Recall: a random variable (rv) X is a function on the sample space of an experiment. (That is, an rv gives a number to each possible outcome.)

Example 1:

(a) If the experiment is "roll two dice," we can define an rv X by

$X(\omega)$ = sum of two numbers on dice in outcome ω.

E.g. $X(13) = 4$, $X(52) = 7$.

(b) If the experiment is "shoot 3 free throws," we might define

$Z(\omega)$ = # of free throws made in ω.

(c) If the experiment is "play the lottery," we could define

$Y(\omega)$ = amount won for a given ticket ω.

Also, given an rv X and a number x, we define $P(X=x)$ to be $P(\{\text{all outcomes } \omega \text{ with } X(\omega)=x\})$.

Example 2 (a) Example 1(a) above, if the dice are fair, we have

$$P(X=8) = P(\{17, 26, 35, 44, 53, 62, 71\})$$
$$= 7/36 \approx 19.44\%.$$

(b) In Example 1(b) above, if the shooter hits 80% of the time, and the shots are independent, we have

$$P(Z=3) = P(\text{all shots made})$$
$$= (.8)^3 = 0.512.$$

New definition: given an rv X, we define the expected <u>value</u> $E(X)$ of X by

$$E(X) = \sum_{\substack{\text{all possible} \\ \text{values of } x}} x\, P(X=x).$$

Note that $E(X)$ is a kind of "expected average" of X.

<u>Example 3.</u>

(a) In Example 1/2 (a) above

$$E(X) = \sum_{k=2}^{12} k P(X=k)$$

$$= 2P(X=2) + 3(X=3) + \ldots + 12P(X=12)$$

write out the outcomes and count each event

$$= 1\cdot\frac{2}{36} + 2\cdot\frac{3}{36} + \ldots + 11\cdot\frac{2}{36} + 12\cdot\frac{1}{36}$$

$$= 7.$$

(b) In Example 1/2 (b), we showed last time that

$$P(Z=0) = 0.002,\ P(Z=1) = 0.096,$$
$$P(Z=2) = 0.384,\ P(Z=3) = 0.512.$$

So

$$E(Z) = 0\cdot 0.002 + 1\cdot 0.096 + 2\cdot 0.384 + 3\cdot 0.512 = 2.4.$$

(c) In the game Wordle, you have 6 guesses to guess a word. Let W be the rv. of

how many guesses it takes to win (assuming a win).
Suppose W has these probabilities:

x	$P(W=x)$
1	0.0002
2	0.0567
3	0.2266
4	0.3310
5	0.2391
6	0.1172

Then

$$E(W) = 1\cdot 0.0002 + 2\cdot 0.0567 + \dots + 6\cdot 0.1172$$
$$= 4.0161.$$

Question: could we have done Example 3(b) above (free throws) without having to know $P(Z=0)$, $P(Z=1)$, $P(Z=2)$, $P(Z=3)$?

Intuitive answer:

The shooter hits at 80%, so on the first shot we'd expect 0.8 shots made out of 1. Same for shots 2 and 3, so we'd expect that, out of 3 shots taken, the number made would be

$$0.8 + 0.8 + 0.8 = 2.4$$

(same answer as before).
Next time: why this works.