Conditional probability:
Complementary events

Let B be any event, and let B^c be its complement (meaning everything in the sample space but B).

Then either B or B^c must happen, but both can't. So, for any event A, to say A happens is to say either AB or AB^c happens, but not both. So

$$P(A) = P(AB \cup AB^c) = P(AB) + P(AB^c).$$

So, by ($*_3$),

$$\boxed{P(A) = P(B)P(A|B) + P(B^c)P(A|B^c)} \qquad (\bowtie)$$

Example. A large jar is filled with 1/3 blue and 2/3 red marbles. 3/8 of the blue marbles are marked with a "W," and 1/4 of the red ones are. You win if you pick a "W" marble.

What is P(win)?

Solution.

A : win
B : draw a blue.
C : draw a red.

Note that $C = B^c$, so by ($\bowtie$),

$$P(A) = P(B)P(A|B) + P(C)P(A|C)$$

$$= \frac{1}{3}\cdot\frac{3}{8} + \frac{2}{3}\cdot\frac{1}{4} = \frac{1}{8} + \frac{1}{6} \approx 0.2917 = 29.17\%.$$

In a population of 10,000 people, 1% are infected with COVID-19. All 10,000 people are tested, using a test that has a 2% false positive rate (2% of those who are uninfected will test positive), and a 7% false negative rate. Complete the table, and the conclusion below.

Number of people	Infected	Uninfected	Total
Test positive	93 (true positive)	198 (false positive)	291
Test negative	7 (false negative)	9702 (true negative)	9709
Total	100	9900	10000

CONCLUSION: Even though the test has a false positive rate of only 2%, out of all people who test positive for COVID-19, only

$$\frac{93}{291} \approx 32\ \%$$ are actually infected!