Conditional probability, continued.

A) Recap.

Recall the formula for $P(A|B)$ (the probability of A given B):

$$\boxed{P(A|B) = \frac{P(AB)}{P(B)}} \qquad (*_2)$$

Example 1.

In shooting a pair of free throws, a player makes the first 80% of the time, and makes both 72% of the time. Given that they make the first, what's the probability that they make the second?

Solution: Write

A: second shot is made
B: first shot is made

Then $P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.72}{0.8} = 0.9 = 90\%$.

B) A twist on $(*_2)$.

Multiply both sides of $(*_2)$ by $P(B)$ to get

$$P(B)P(A|B) = P(AB),$$

which is more useful switched around:

$$\boxed{P(AB) = P(B)P(A|B)} \qquad (*_3)$$

Example 2.

In shooting a pair of free throws, a WNBA player makes her first shot 85% of the time. Whenever she makes her first, she makes her second 90% of the time. What's the probability she makes both?

Solution

If A and B are as in Example 1, then by $(*_3)$,

$$P(AB) = P(B)P(A|B)$$
$$= 0.85 \cdot 0.90 = 0.765 = 76.5\%$$

c) Generalization of $(*_3)$.

Given events A, B, C, it follows from applying $(*_3)$ twice that

$$\boxed{P(ABC) = P(C)P(B|C)P(A|BC).} \qquad (*_4)$$

Similarly, for four events,

$$\boxed{P(ABCD) = P(D)P(C|D)P(B|CD)P(A|BCD).} \quad (*_5)$$

And so on.

Example 3.

Four cards are drawn at random from a standard deck. Find P(all four are clubs).

Solution.

P(all are clubs)

= P(first is and second is and third is and fourth is)

= P(first is)$\cdot$P(second is, given first is)

$\cdot$P(third is, given first two are)

$\cdot$P(fourth is, given first three are)

$$= \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \approx 0.0026 = 0.26\%.$$

Question: using this idea, find
P(a flush)
(five cards of the same suit), when five cards are drawn at random from a standard deck.

(D) Suppose A and B are independent events. Then whether B happens doesn't affect whether A does. So $P(A|B) = P(A)$. So $(*_3)$:

$$P(AB) = P(B)P(A|B)$$

becomes

$$P(AB) = P(B)P(A),$$

which we already knew about independent events.