Solutions to some probability questions.

Question 1: What's the probability that, of the 23 people in this class, at least two share a birthday? (Assume there are no Feb. 29 birthdays.)
Is this probability (A) $< 50\%$, or (B) $\geq 50\%$?

Solution.

First, we compute the probability that all 23 birthdays are different. Like this:

(a)(i) Pick Person 1. The prob. that they don't share a birthday with anyone else picked so far is $365/365$.

(ii) Pick Person 2. The prob. that they don't share a birthday with anyone else picked so far is $364/365$.

(iii) Pick Person 3. The prob. that they don't share a birthday with anyone else picked so far is $363/365$.

⋮

(xxiii) Pick Person 23. The prob. that they don't share a birthday with anyone else picked so far is $(365-22)/365 = 341/365$.

(b) So the prob. that no two share a birthday is

this numerator is denoted ${}_{365}P_{23}$ ("365 pick 23")

$$\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdots\frac{341}{365} = \frac{{}_{365}P_{23}}{365^{23}} \approx 0.4927.$$

(c) So the prob that at least two share a birthday is $\approx 1 - 0.4927 = 0.5077 > 50\%$.

Question 5: In a Powerball lottery, where you must match 6 out of 45 different numbers exactly, is your probability of winning

(A) greater than; or (B) less than

the probability of a fair coin, flipped 23 times, landing heads each time?

Solution.

There are $\binom{45}{6}$ ways of choosing 6 numbers out of 45. There's only one way of matching all six exactly. So the prob. of winning this lottery is

$$\frac{1}{\binom{45}{6}} = \frac{1}{8,145,060} \approx 0.000000123 = 1.23 \cdot 10^{-7}.$$

On the other hand, for the coin toss, there's prob. ½ of heads landing each time, so the prob. of 23 heads in 23 tosses is

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \cdots \cdot \frac{1}{2} = \frac{1}{2^{23}} \approx 0.000000119 = 1.19 \cdot 10^{-7}.$$

So winning this lottery is slightly more likely.

Question 8:

You wash 10 pairs of socks, each pair a different color, and after washing, 6 socks are lost. Which is more likely:

(A) The best case scenario (seven matching pairs left); or

(B) The worst case scenario (four matching pairs left).

Solution:

There are 20 socks total, so there are $\binom{20}{6}$ ways of losing six socks. Now:

(A) There are 10 pairs, so there are $\binom{10}{3}$ ways of losing exactly 3 pairs. So the prob. of the best case scenario is

$$\frac{\binom{10}{3}}{\binom{20}{6}} \approx 0.0031 = 0.31\%$$

But:

(B) How many ways are there of losing 6 socks all of different colors? Well, there are $\binom{10}{6}$ different ways of choosing the 6 lost colors, and for each of these 6 colors, there are 2 choices of which sock to lose. So there are

$$\binom{10}{6} \cdot 2^6$$

ways of losing 6 different-color socks.
So the prob. of doing this is

$$\frac{\binom{10}{6}\cdot 2^6}{\binom{20}{6}} \approx 0.3467 = 34.67\% .$$

So the worst case scenario is <u>much</u> more likely.