Some Additional Review Problems

Here we consider two similar scenarios. For both of these, recall that a standard deck of cards has 4 aces, and 48 cards that aren't aces.

(a) We draw three cards *without replacement* at random from a standard deck. That is: each card that is drawn is left out of the deck before drawing the next card. (This is the usual way cards are drawn or dealt.) Let X be the number of aces drawn.

(b) We draw three cards *with replacement* at random from a standard deck. This means: We draw the first card, record what kind of card it is (ace or not), and *put it back in the deck*. We then draw the second card, record what kind of card it is (ace or not), and put it back in the deck. Finally, we draw the third card, and record whether it's an ace. Let Y be the total number of aces drawn.

Here are the questions.

1. One of the above scenarios represents three independent trials of a binomial experiment, and one does not. Which is which? Why?
2. Let's look at X (from scenario (a) above) first. We will compute $P(X = 1)$ by *keeping track of order*, as follows.
   (a) Find P(the first card is an ace and the other two are not).
   (b) Find P(the second card is an ace and the other two are not).
   (c) Find P(the third card is an ace and the other two are not).
   (d) Find $P(X = 1)$ by adding.
3. It's easier to find probabilities for X *without keeping track of order*. Let's do this: compute the probability mass function for X. That is, compute $P(X = k)$, for $k = 0, 1, 2, 3$. Hint: in each case, consider: (i) How many ways are there of drawing 3 cards out of 52 (without keeping track of order)? (ii) How many ways are there of drawing k aces out of 4 (without keeping track of order)? (iii) How many ways are there of drawing $3 - k$ aces out of 48 (without keeping track of order)?

   Note: in the case $k = 1$, your answer should agree with Exercise 2 above.
4. Compute $E(X)$, $\text{var}(X)$, and $\text{std}(X)$.
5. This time, compute the probability mass function for Y (from scenario (b) above).
6. Compute $E(Y)$, $\text{var}(Y)$, and $\text{std}(Y)$.
7. How do $E(X)$ and $E(Y)$ compare?
8. Which is larger, $P(X = 0)$ or $P(Y = 0)$? Why does this make sense? That is, could you have predicted this without any computation, and if so, how?