Homework Assignment #5: Due Monday, October 23

Please read Apostol, Sections 6.4–6.10. Then please do the following exercises:

Part I: Apostol Chapter 6, pp. 143–145. Exercises 13, 15, 17.

Notes on Exercise 13: (i) Although it's not specified in the exercise, you should assume that G is abelian. (ii) It might actually be better to look at the sum

$$\sum_{r=1}^{m}\sum_{k=1}^{n} f_r(a^k)e^{-2\pi i\ell k/n},$$

where ℓ is an arbitrary integer between 0 and $n-1$.

Part II. (A) Evaluate (as a real number) the series

$$L(1,\chi) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n},$$

where χ is the unique nontrivial Dirichlet character mod 3. Hint: after writing the series out explicitly, consider the integral $\int_0^1 t^{3n}(1-t)\,dt$. Note that tables of values of Dirichlet characters are given on p. 139 of Apostol.

(B) Repeat **(A)** above for the unique *real-valued*, nonprincipal Dirichlet character mod 5. The series should end up as an integral of a rational function. Do the best you can with this integral: leave as is, evaluate numerically or, if possible, evaluate explicitly.